PRACTICAL ACOUSTICS

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LESSONS

IN

ELEMENTARY PRACTICAL PHYSICS

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IN

ELEMENTARY PRACTICAL PHYSICS

STEWART AND GEE SERIES

BY

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PRACTICAL ACOUSTICS

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PREFACE

THIS book forms the first part of Volume III. of the "Elementary Practical Physics" series begun in 1885 at Owens College by Professor Balfour Stewart and Mr. W. W. Haldane Gee. Upon the lamented decease of the former in 1887, and the subsequent acceptance by Mr. Gee of an appointment as Chief Lecturer in the electrical department at the Manchester Technical School, the work was allowed to lapse for some years. Ultimately it was decided, with the concurrence of the Publishers, to entrust the remaining sections to separate coadjutors. That on Heat is in preparation by the general Editor, to whom the present writer is indebted for help and suggestions: the final volume (Optics) will appear later.

In the following pages are collected most of the experiments in Acoustics which can be performed with the usual appliances at the command of a student; others of a more elaborate character are described or referred to, though not numbered as

part of the course. More than one experiment is sometimes included under the same heading in order to avoid a needlessly minute subdivision.

In the other volumes of the series the wealth of quantitative experiments has marked out the broad lines upon which the work should proceed; but in Acoustics, where this feature is lacking, some alteration in plan is inevitable: hence this volume is in some respects a general text-book.

The order of the experiments has not been settled without difficulty: probably there is no arrangement against which some more or less serious objection might not be urged. To begin by discussing the nature of Harmonic Motion in detail usually results in the student's being recommended to omit it on a first reading; to delay it too long is to keep in the background the fundamental principle of the science. It is hoped that the middle course here adopted will prove satisfactory.

A number of references to original papers in the *Philosophical Magazine, Nature,* and elsewhere, are given, so that those who have the volumes at command will be able to supplement what is here presented. No one can now or hereafter write a book on Sound worthy of the name, without being under extensive obligations to Lord Rayleigh's great work; a free acknowledgment of the assistance derived from this source is here offered.

Many of the figures are original, some have been taken, by the kind permission of Dr. Koenig, from his *Catalogue des Appareils d'Acoustique*, the remainder are from well-known sources.

In Appendix IV. a list of names of workers in theoretical or experimental Acoustics is given, with dates of birth and death. Only in some such way as this can any idea of the perspective of things be gathered: it is avowedly imperfect, but to prepare an exact chronological table is impossible, and any more serious attempt appeared not to be worth the extra trouble it would involve.

The author is well aware that a book of this kind cannot be presented in its best or most permanent form at a first attempt, and will be glad to receive suggestions on any point from those engaged in teaching the subject. These will be carefully considered if a second edition should be called for.

MANCHESTER, *2nd April* 1897.

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PRACTICAL ACOUSTICS

CHAPTER I

SOUND

THE word Sound is used to express two different ideas: one the sensation of hearing, the other the cause of this sensation. In the science of Optics the corresponding ideas are denoted by the two words vision and light, but in Acoustics, when we wish to emphasise the distinction, we are obliged to speak of the sensation or the source of sound respectively.

Generally the latter meaning alone is intended, and often in the restricted sense of a musical sound as opposed to a noise.

That a sounding body is in a state of vibration may be proved in many ways: with a stretched string or a tuning-fork it is visible to the eye, but when the extent of vibration is very small it can still be detected by the touch or by its action on light bodies, such as grains of sand, etc.

EXPERIMENT I

To show the Symmetry of Vibration in a Body emitting a Musical Sound

Required.—A large evaporating basin, about half full of water; resined bow; ether.

Let the basin rest on a suitable support, say a metal ring or an iron triangle covered with cloth: draw a bow down the edge. If necessary, press the basin in the middle with a wooden rod to keep it steady. Observe that the surface is thrown into a multitude of wavelets which spring from four equidistant points in the circumference. By forcible bowing the water may be caused to spring up in drops, but they are much more easily formed when ether is poured over the surface; because this liquid is immiscible with water, and is not only considerably lighter than it, but has a smaller surface-tension.

In other cases, also, the sounding body undergoes a symmetrical distortion, *e.g.* a bell, rod, or plate.

For a sound to be communicated to the ear some elastic medium is necessary, either the air or a mass of wood, water, metal, etc., or some combination of these. The transmission is effected by means of waves, the investigation of whose forms, velocities, superpositions, energy, etc., is a branch of Hydrodynamics, and is quite independent of the physiological and aesthetic aspects of the subject; in other words, it is of no consequence to the theory whether such waves excite the sensation of hearing or not, still less whether they do so agreeably.

As with other forms of energy, the ultimate destiny of sound waves is to be converted into heat.

Experiment II

To compare the Transmission of Sound in various Materials

Required.—Rods of wood, metal, ebonite, glass, etc.; a wax candle; piece of rubber tubing; about half a metre of fine wire; tuning-fork.

Place one end of each of these materials in contact with the ear; sound the tuning-fork (it should not be struck on a hard surface) and press it on the other end.

As a rule the sound will be heard clearly, even at distances where it is inaudible in air; with string, paper, and similar substances this is not the case until they are stretched. The experiment may be varied by pressing the fork on one end of each rod in turn, while the other end is touching the resonance box.

Observe if there is any difference between the conductivity of a fine wire for the proper note of a fork and the jangling noise it gives out when struck with a hard body.

Although a fork or a watch cannot be heard beyond a short distance in air, it must not be concluded that air conducts so much worse than other substances, for when confined in a tube, so that the waves are prevented from spreading (as they are also in rods or wires), a low sound can be heard over a great distance, as in a speaking-tube. In general the conductivity is best in those materials in which the rate of propagation is greatest. Indiarubber conducts very badly: though commonly known as "elastic" its elasticity is really very small, and the property it possesses in so high a degree is quite a different one, viz. extensibility

Experiment III

Conduction through Water

Required.—Beaker of water; tuning-fork; small block of wood.

Place the beaker on an uncovered table, rest one ear against it, and dip the shank of the vibrating fork in the water: it will be heard very faintly, if at all. Next dip a prong in and observe how much louder the sound becomes. Take a block of wood, small enough to go into the beaker, drill a hole in it to admit the stem of the fork, and repeat the experiment. The greater surface exposed now removes

the disadvantage attaching to a thin shank, and the sound is heard plainly. As before, it may be conducted to a resonance box by an obvious arrangement. Other liquids may be substituted for water, and it is a simple variation to try an effervescing one by using dilute hydrochloric acid, and adding some carbonate of soda.

Let a steady stream of water flow from a tap upon the wooden block, and by putting the ear to the supply pipe, find whether or not the sound is conducted upwards against the current.

Experiment IV

To show that Sound can be cut off by an Obstacle

Required. — Sheet of cardboard; watch; wide glass cylinder; rubber tubing; fish-tail flame.

Hang the watch to a retort-stand or other support; hold the sheet of cardboard in a vertical position and move the ear to various points on the opposite side. It will be observed that a sound-shadow is produced, within which the ticks are inaudible. Now hold a book beyond the edge so as to reflect the waves into the ear: they will be heard once more.

Lower the watch into the cylinder; it will be easily heard all round. Now let it rest on a coil of rubber tubing instead of on the glass, and the ticks can only be heard within a certain space round the top. It is of course notorious that ordinary sounds can pass round corners easily enough, but the waves excited by a watch being small and feeble, they are cut off as we have said.

Place the watch on one side of a large fish-tail flame, and observe that the ticks are heard very indistinctly, if at all, on the other side. It may be necessary to put a cloth round the watch to deaden the sound slightly.

Experiment V

To show that Sound cannot pass through a Vacuum

Required.—Air-pump and receiver; small loud-ticking clock; a cushion or coil of rubber tubing.

Lay the clock on the non-conducting material and cover it with the receiver: it will probably be heard distinctly outside. Now exhaust the air: the ticks will become fainter, though audible to an ear pressed against the glass. This result is due to the lessened energy with which the sound waves strike against the glass. It is natural to infer that in a perfect vacuum no sound whatever could be heard. On readmitting air the original state of things is restored. The experiment was first performed by Boyle about 1662.

Experiment VI

Effect of admitting Hydrogen

Required.—A hydrogen cylinder or large apparatus for the supply of the gas; air-pump, etc., as before.

Turn off the stopcock underneath the receiver to prevent loss by diffusion. Slide the receiver till it projects slightly over the edge; admit hydrogen rapidly through a tube reaching nearly to the top of the receiver, till it may be considered full; then restore it to its place. Although there is now not even a partial vacuum, the sound is practically extinguished, and exhausting the gas will produce no appreciable difference.

This phenomenon was known to Priestley, but remained long unexplained. It is due to the greater lightness and also to the mobility of hydrogen, by which a large vibrating surface can pass through it without exciting waves, the pressures being rapidly equalised by a flow on either side. So, also, a wire stretched between two

supports, but not over a resonance case, gives out little or no sound when plucked, because it slides through the air without disturbing it. On the other hand, it is scarcely possible to touch a tuning-fork so lightly as not to make it ring: here the broad surface, though perhaps not much greater than that of the wire, excites the air readily. Again, a very fine wire may be pulled rapidly across a surface of water and hardly disturb it at all, while a thick one sets up waves at every point.

Experiment VII

Production of a Vacuum by Condensation

Required.—Round-bottomed flask; perforated rubber stopper; glass tube drawn out at one end, and having a hole in the side as at A, Fig. 1; short rubber tube; and a toy bell.

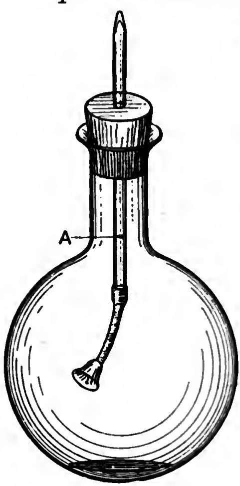

Fig. 1.

Pour a little water in the flask, and arrange as in the figure. Boil the water till no air is left inside; seal up the tube by holding it in a Bunsen flame, and allow to cool. On shaking the bell it cannot now be heard. Heat the water again and it is audible once more. Care must be taken to control the violent "bumping" which occurs even at a low temperature, owing to the strong adhesion of water and glass when no air is present. Alcohol may be used instead of water in the experiment: ammonia answers very well, but soon attacks the bell and turns blue by dissolving the copper.

Experiment VIII

Transmission of Speech through String.

Required.—Two shallow cardboard boxes and a length of string.

Make a small hole in the bottom of each box, and tie a knot on the string so that it will not slip through. Stretch the string, which may be 20 metres or more in length, and let one observer speak into his box while a second listens at the other end: the sounds of the voice are transmitted well enough to be recognised. The apparatus is called a string telephone. Iron or copper wire may also be used, but the bottom of the cup must then be of thin metal, to which the wire is soldered.

Experiment IX

Sonorousness of different Materials

Required.—Rods of steel, glass, aluminium, wood, etc.; also bell-shaped masses of these or other elastic substances.

Hang up each mass by a thread and tap it with a finger or a pencil. Notice the duration and loudness of the sound; also that anything shaped like a bell rings much better than a block or even a straight bar.

Aluminium, though noted for its sonorousness, has lately been found by Prof. Mayer[1] to be unsuitable for bells, tuning-forks, etc. It is very readily excited, and gives a louder sound than most other metals, but the duration is very short owing to imperfect elasticity: its properties also vary too rapidly with the temperature for it to be of much use.

Experiment X

Sound and Noise

Required.—Sonometer; several organ pipes; tuning-fork; ordinary hair comb; piece of cardboard.

[1] *Phil. Mag.* vol. xli. (1896).

Pluck the sonometer wire, and it gives a resonant sound of some duration. Place two or three fingers so as to stop the wire immediately after plucking: no distinct musical sound will now be heard, but only a noise. Organ pipes when blown of course give a continuous sound, but when tapped it might hastily be concluded that they emitted nothing but a noise. On tapping several, one after the other, it is quite evident, however, that each has a pitch of its own. To show that this is due to the air inside, fill a tube loosely with cotton wool, then whatever sound it gives on being tapped is due to the wood or metal alone. Strike a tuning-fork in the ordinary way and it will give its particular note, but take hold of the prongs about half-way up and pluck it: no note can be distinguished. Draw a card slowly over the teeth of a comb, the successive taps are heard as noises: draw it more and more quickly, and they blend into a sound which, if not exactly musical, is yet sufficiently so for a rise in pitch to be noticed as the speed is increased.

We conclude, then, that a musical sound may be cut off so sharply as to strike the ear as a noise; and, on the other hand, that a series of noises, if they follow each other rapidly enough, become musical. The ear is ever ready to detect harmony in the rustling of leaves or the sound of a waterfall on the same principle.

The number of impulses necessary to give the sensation of a musical note is very small, five or even less being sufficient.

Differences between Musical Sounds

It is a matter of common experience that musical sounds may differ from each other in three ways, viz. in loudness, pitch, and quality. The first of these is due to the extent of vibration, as is well shown by a tuning-fork, whose tone becomes fainter as the swing of the prongs

diminishes. The manner in which intensity and amplitude[1] are connected is explained on p. 140. It is hardly less obvious that the *pitch* of a note, *i.e.* its position in the musical scale, is governed by the rate of vibration, and lastly the *quality*, or property by which we distinguish the sounds of different instruments, depends on the nature of the waves they set up. A tuning-fork, for example, sets up waves of one kind, a string waves of another kind, in an organ pipe they differ from both, and so on. This subject is discussed more fully in the chapter on Harmonics (p. 150).

[1] Technically the amplitude is half the extent of swing.

CHAPTER II

NATURE OF WAVE-MOTION

A WAVE is the usual means by which energy is transmitted from one point of an elastic medium to another. Thus when a stone is thrown into water, or a distended paper bag is burst by a blow, a disturbance is created which is propagated in circular or spherical waves to other points around. There is no transference of matter, for that would require a current. In a solid, such as a long bar of iron, a tap with a hammer at one end displaces the particles upon which the blow falls, and from them it is communicated to the adjacent particles in the form of a wave till it reaches the other end. If a ball were hung by a thread so as just to touch the bar at this point, it would be knocked away even though the bar itself were firmly clamped. Some of the principles of wave-motion are discussed from an elementary standpoint in Chapter IX., but we may remark here that the waves concerned in the propagation (not the production) of sound are essentially the same in all classes of media whether solid, liquid, or gaseous, and consist in a periodic condensation and dilatation of the particles.

EXPERIMENT XI

Progressive and Stationary Waves

Required.—Two or three metres of rubber tubing or soft rope.

Hold one end in the hand while the other parts lie on the floor. Shake it rapidly backwards and forwards: a series of *progressive transverse* waves is thus set up, whose energy is lost in friction; hence no motion is returned from the far end. Such waves are called "transverse," because the movement of each particle is at right angles to the direction in which the wave is travelling. Observe that the more quickly the hand is moved, the shorter and more numerous are the waves, not because they are propagated more quickly, but because there are more of them in the same space. Next let the other end be held by a companion, who also shakes it, so that impulses are sent in both directions at once while it is clear of the ground. At first the result will probably be confusion, but by a mutual agreement which soon automatically suggests itself, the whole tube is thrown into a regular oscillation, and illustrates what are called *stationary* waves. In some of the succeeding experiments the production of these waves is the object in view, but it is not always obvious that a meeting of impulses from both ends is necessary. For example, hold the tube vertically at such a height that it does not touch the ground, shake the upper end backwards and forwards: it is still possible to excite stationary waves. Here a reflection takes place from the free end (because the energy cannot escape), which is no less efficacious than if some one shook it. A stretched string or wire corresponds exactly to one or more sections of the tube, and its condition is maintained in the same way by the meeting of waves in opposite directions (see p. 96).

Experiment XII

Different Types of Waves. I. Transverse

Required. — Helix of brass wire, and support for the same, about 2 m. from the ground. The following dimen-

sions are suggested, but need not be closely adhered to. The wire, No. 15 gauge, is 6 m. long, and is coiled round a tube 3 cm. in diameter, and adjusted to a length of 1½ metres.

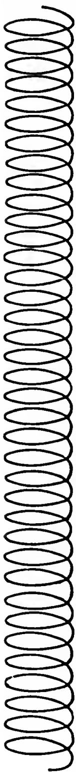

FIG. 2.

The coil being hung in a vertical position, take hold of the lower end, and give it a quick shake sideways: a ridge will then form, run rapidly up to the top, and be reflected down again. Observe that it comes down on the opposite, not on the same side, obeying the familiar law that the angle of incidence is equal to the angle of reflection. Also that when the coil is pulled out to a greater length, the velocity of propagation, which depends on the elastic recovery of the wire, is much increased. When the lower end is held firmly, and the coil plucked, reflections take place in the same manner both above and below, but if it be not held, then the lower end sways from side to side, and a stationary point forms about a third or a fifth of the way up. By an instinctive motion of the hand, it is possible to keep the coil vibrating in one or in many sections as in Expt. XI.: these as before are so many halves of a stationary transverse wave. When the hand is moved irregularly, the direct and reflected systems interfere with one another so that stationary waves are not set up.

Two successive sections, or as we may term them, a ridge and a furrow, constitute a complete wave, just as a wave in water is composed of a crest and a trough.

The stationary points are called *nodes*, the vibrating portions *loops*, or ventral segments; the middle of a loop is an *antinode*, because here the excursions on either side are greatest, the converse of what takes place at a node.

The distance between two consecutive nodes or antinodes is obviously half a wave-length. The time in seconds of a complete vibration to *and* fro is known as the *period*, and the number of vibrations in a second is the *frequency*, so that period and frequency are reciprocals of one another. The displacement of an antinode from its initial position is the *amplitude*. It is clear that the mean velocity of each particle is proportional to its own amplitude.

Experiment XIII

Types of Waves. II. Longitudinal

Now give the coil a light but sudden push from beneath, so as to compress several of the rings together. This compression can be seen to travel up and down as before, but when it reaches the bottom, this end sinks, and since there is no reaction such as takes place from a fixed end, the condensed pulse becomes a dilated one, and travels up and down as such.

This illustrates *longitudinal* vibration, where the motion of each particle is to and fro in the direction in which the wave is travelling. By holding the lower end and moving it in accordance with the impulses received, a series of stationary longitudinal waves can be set up, in which the words node, antinode, period, etc., have the same general meaning as before. From a fixed end, whether above or below, a condensation is reflected as a condensation, and a dilatation as a dilatation, *i.e.* there is no change of type, but only of direction; from a free end there is a change both of type and direction, *i.e.* a condensation travelling downwards is reflected as a dilatation travelling upwards, and conversely. This is a very important principle in connection with resonators and organ pipes.

Experiment XIV

Types of Waves. III. Torsional

Lastly, give the lower end a sudden twist, so as either to tighten or slacken the coiling: a *torsional* vibration is now produced, and is reflected up and down after the manner of the other two.

In all these cases the movement is (at least approximately) governed by a certain law, viz. that the force of restitution or recovery is proportional to the displacement, provided that the latter does not exceed a certain limit. Suppose, for instance, that a weight of 20 grs. lengthens the coil 2 cm., then 40 grs. will lengthen it 4 cm., and so on. This is the principle of the ordinary spring balance. Again, if it takes a certain force to twist the end of a coil so many degrees, twice that force will twist it through double the number. Here we have the principle of the torsion balance, viz. that the angle of torsion is proportional to the force of torsion. Finally, if the middle of the coil be pulled aside while its ends are fixed, a similar relation holds. In all cases the motion is analogous to that of a simple pendulum, because here also within small limits the displacement is proportional to the force applied. The importance of this principle will be more evident when we come to deal with Harmonic Motion (Chapter IX.).

A wire stretched between two supports, but not coiled, comports itself in the manner already described, but its vibrations are now very minute and rapid, so that only transverse vibrations are visible to the eye, but the longitudinal and even the torsional are capable of affecting the ear.

Waves in Air

The condition of the air under the influence of a simple sound-wave, *i.e.* one produced by a pendular movement

such as has been described, is analogous to the foregoing, but being a gas it can only vibrate longitudinally. Variations of pressure take place in it, and would be capable of affecting a barometer if they were not too local and rapid. The condition of things is well illustrated by the following apparatus.

Crova's Disc

On a circular disc of cardboard 31 cm. in diameter draw a circle 5 mm. in diameter concentric with the disc, and divide it into 12 equal parts. From one of these points of division as centre draw a circle of 7·5 cm. radius, from the next point a circle of 7·8 cm. radius, and so on, adding 3 mm. each time and going twice round so that there are 24 circles in all, as shown in Fig. 3. Mount the disc on a multiplying wheel arrangement, and hold in front of it a piece of cardboard in which a narrow rectangular slit has been made. On viewing the circles through this hole, they alternately close up and recede from one another in the manner of air particles in a progressive wave. By reversing the direction of rotation, the movement takes place from without inwards.

Free and Forced Vibrations

A body which is in a state of vibration may continue to move in a natural period of its own, depending on some force of attraction or elastic recovery, as for instance a pendulum, a balanced compass needle, or a stretched wire set in vibration and left to itself,—or it may be obliged by external means to take up a vibration differing from this in form, or period, or both. In the former case its vibration is said to be *free*, in the latter *forced*. A succession of impulses given at the proper time, *i.e.* at the middle of the swing, may, however, exactly supply the loss by friction or other resistance, so that the motion,

though continuous, is still free. A clock pendulum does

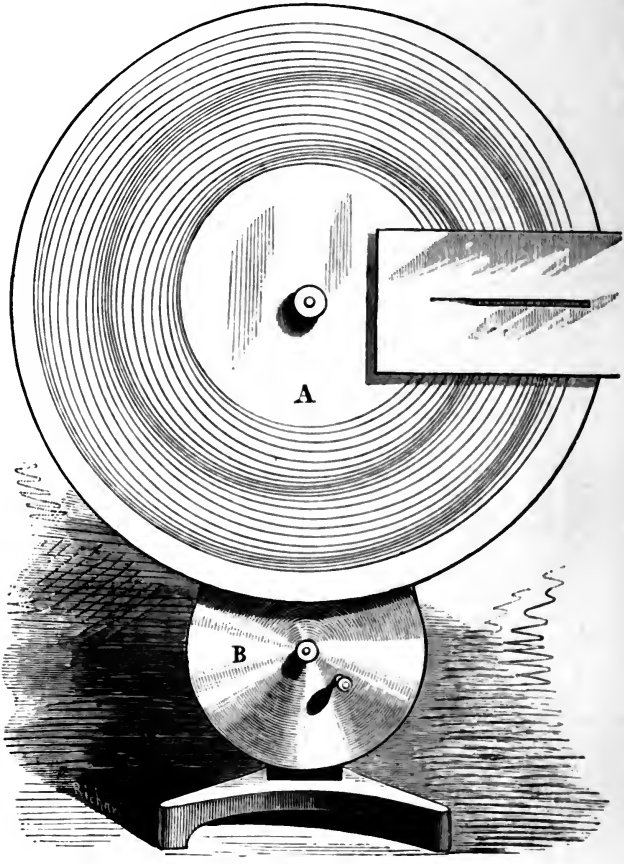

FIG. 3.

not move solely under the action of gravity, but slightly

quicker owing to the pressure of the pallets; if these acted at the *middle* of the swing only, the oscillation would be free and of exactly the same period; as they do not, it is slightly forced. The waves raised on water when a stone is dropped in are free, those excited by a gale are forced. The vibration of a tuning-fork when excited by an electro-magnet is forced, and the period is not quite, though very nearly, the same as if it were left to itself (see p. 79). Again, in a reed pipe, the air executes forced vibrations, being obliged to follow the motion of the reed.

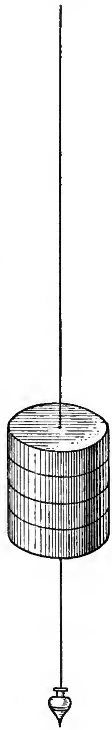

Fig. 4.

Experiment XV

Illustration of Forced Vibration

Hang a weight — the heavier the better — to a wire or cord 1 m. long or more, and underneath it fasten a small one, say a bullet, by a flexible thread. Set the big weight swinging through a very small arc, and observe that the motion of the lower pendulum is quite different from what it would be under ordinary circumstances; it is in fact a forced movement. Naturally the upper pendulum is also affected by the lower, but in a much less degree: it comes to rest much sooner than if the extra weight had been laid on the top or rigidly fastened to it in any way.

CHAPTER III

THE SONOMETER

THIS is one of the most useful and valuable instruments in Acoustics; it consists (see Fig. 5) of a wooden box rather more than a metre long, over which two wires are stretched between wrest-pins, as in a piano; these can be turned in either direction by means of a tuning key. In the best instruments the pins are prevented from tearing the wood

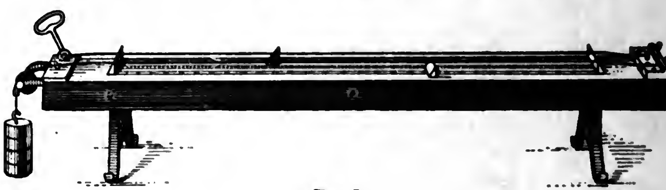

FIG. 5.

by a small plate of brass, which is bored to receive them. A wooden bridge at each end defines the limits of vibration of the "open" wire, and through these, which are at nodal points, the sound is conveyed to the box. A small metal slide, provided with a notch, can be placed at any point, creating a node there without altering the tension. A triangular wooden prism is not quite so good. A brass pulley projecting over the end of the box is sometimes provided, over which a third wire can be stretched by means of weights; but its rigidity or resistance to bending

diminishes the tension by an unknown force, and it is better for numerical purposes to use a spring balance. There is usually a metre scale between the wooden bridges, but where accuracy is required, the lengths must be taken by a beam compass or trammels (Fig. 6). The use of this is almost obvious on inspection. Two blunt steel needles may be clamped anywhere on the wooden bar, and when they are separated to the required distance they are brought down on a metre scale, and the space between them is read off.

The object of the box, which is sometimes dispensed with—though at a great sacrifice of utility for anything like delicate effects—is to increase the intensity of the sound; this it does both by exposing a large vibrating surface, and by confining a volume of air which, as in

FIG. 6.

stringed musical instruments in general, has a considerable range of sympathetic vibration. It is generally necessary to clamp the feet of the box on the table, or to place a thick piece of cloth underneath to prevent jarring.

The wires are excited either by drawing a resined bow across them, or by plucking with the fingers (see Appendix I.), or occasionally by a blow with a padded hammer. For very delicate work the fingers must not be used, owing to the warmth which they communicate.

The method of excitation with a bow depends on a well-known mechanical principle, viz. that the friction between two bodies at rest is greater than when they are in relative motion. The resined hairs lay hold of the wire and pull it from its original position: a limit is shortly reached, and it springs back again, now passing the hairs freely according to what we have said; the motion is then again reversed, and for an instant the bow and wire are

relatively stationary (though both actually in motion), the gripping now takes place once more, and so on. In precisely the same way, by hanging a small weight tied to a piece of thread, with a loop at the top which is passed round a glass tube (see Fig. 7), a pendulum motion may be

FIG. 7.

got up by rotating the tube in *one* direction only, the friction as before being greater when the tube and loop are stationary with respect to one another, and less when there is relative movement.

In whatever way a wire is excited, its motion is far from simple: it executes transverse vibrations of many

orders, and since it cannot be pulled out of the straight without also being stretched, a longitudinal motion is set up (*i.e.* waves of compression and dilatation pass along it from end to end) and it also executes torsional vibrations. Except with very thick wires, however, the latter give rise to no audible effect, nor do the longitudinal ones, at least not if the bow is drawn properly; and of the transverse vibrations, that due to the backward and forward swing of the whole wire is in general much more pronounced than any of the minor ones, and is called its fundamental note. As to these minor motions, we shall have more to say hereafter, but some idea of their number can be gathered by stretching a wire several metres long between two suitable supports, and watching the waves which chase one another up and down when it is plucked.

To hold an incandescent electric light bulb in the hand and watch the filament is also a useful lesson in the reflection and superposition of small waves.

Experiment XVI

Laws of the Vibration of a Stretched String or Wire
I. Variation with Length

Required. — Sonometer with movable bridge or slide; resined bow; several tuning-forks of known rates of vibration; beam compass; metre scale.

Tune one of the wires to the fork which gives the lowest note. To do this, make a "rider" by folding a small rectangular piece of paper into a V-shape, and put it on the middle of the wire. Adjust the tension till it falls off almost instantly when the shank of the (vibrating) fork is pressed on the bridge or the box itself. If the rider is merely agitated and not thrown off the unison is not perfect. Measure the length of the wire from end to end, and note it down. Next, leaving the tension un-

altered, move the bridge till the vibrating portion of the wire is in unison with the second lowest fork, as tested in the same way. Note the length and rate of vibration, or *frequency* as it is usually called, and compare the results. This is best done on squared paper, as shown in Fig. 8. In an actual case the lengths of wire and corresponding frequencies were respectively as follows: 100 cm., 256: 79·9 cm., 320 : 67·0 cm., 384 : 50·05 cm., 512. Dividing

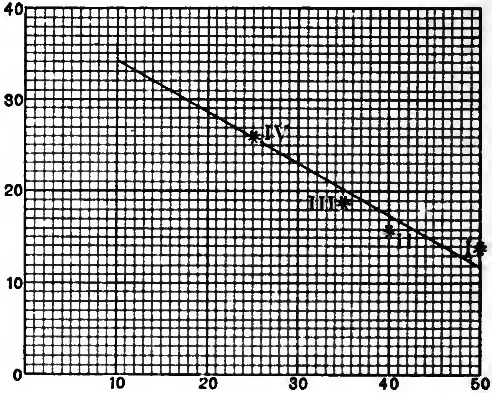

Fig. 8.

the lengths by 2 and the frequencies by 20 we obtain the points I, II, III, IV, which approximately lie on a straight line. This shows that length and frequency vary inversely as one another. Dividing by any constant numbers, as we have done, merely alters the position and inclination of the line on the paper, and in no case does it alter its nature. Neither does it matter if the frequencies are set off horizontally and the lengths vertically.

We may arrive at the same result in another way.

Observing that 320 : 256 = 5 : 4, while 79·9 : 100 = 4 : 5

nearly, the inverse variation is suggested. Calculating what the lengths should be on this supposition, we have

Frequency.	Length (observed).	Length (calculated).	Error.
256	100	...	...
320	79·9	80·00	– ·1 cm.
384	66·64	66·66	– ·02
512	50·05	50·00	+ ·05

The determination is subject to the following sources of error :—

I. Imperfect tuning. This with care need only be very small.

II. Error in estimating the lengths, or imperfect stopping by the bridges. This can be reduced by taking the mean of several determinations, and using bridges which are not too blunt.

III. Alterations of temperature. These, though small, are practically unavoidable. According to Scheibler, "the string cannot be protected from the warmth radiated by the observer's body, even when it is so thoroughly covered that there is only just space enough left for striking it. The string of a monochord from this cause does not remain for 30 seconds at the same pitch, but varies constantly by $\frac{1}{10}$ to $\frac{1}{2}$ of a double vibration." [1]

The mathematical formula which expresses the above and other relations is given after Expt. XIX.

Experiment XVII

II. Variation with Tension

Required.—Sonometer ; spring balance, reading to about 25 lbs.

The balance is placed horizontally and is attached to one of the wires. It is hooked on to a screw at the other

[1] Quoted from Stone's *Sound*, p. 85.

end in the specially-arranged instrument; and there is only one fixed bridge. The tension is adjusted by turning the pin as usual. In the ordinary form of sonometer it will be advisable to place a block of wood containing an upright screw on one end of the box, thus prolonging the top to a convenient length. The tension will maintain it in position. Set the tension at say 20 lbs., measure the length of wire vibrating, tune the other wire to the same pitch, and keep it as a standard for all the experiments. Now lower the tension slightly, move the slide till the note is the same as before, read off the length, and continue the operation a number of times. Since the note does not alter, we have only two quantities varying, viz. length and tension. According to theory l varies as the square root of τ, or in a formula $l = c\sqrt{\tau}$, where c is a constant. We can eliminate c by taking two values l, l', and the corresponding tensions τ and τ'; we thus obtain $\frac{l}{l'} = \sqrt{\frac{\tau}{\tau'}}$. And knowing any three of these quantities, we can calculate the fourth.

The following are some results obtained :—

	Tension (lbs.)	Observed Length (cm.)	Calculated Length.	Error
I.	17·50	57·2		
	14·25	52·0	51·6	+ ·4
II.	15·20	57·2		
	13·75	54·7	54·34	+ ·36
III.	11·25	47·55		
	10·75	46·60	46·46	+ ·14
	9·80	44·50	44·64	− ·14
	8·0	39·75	40·08	− ·33
	7·0	37·35	37·47	− ·12

Nos. I., II., and III. were of course tuned to different pitches. The first calculated length, 51·6, is $57{\cdot}2\sqrt{\frac{14{\cdot}25}{17{\cdot}5}}$ and the others are obtained similarly. It is impossible to get very good results with spring balances, as they do not read accurately enough. By hanging a wire vertically and

providing it with a sounding board at the top, and using precautions to cut off the vibrations sharply at definite points, several sources of error are eliminated, but for ordinary purposes such refinements are needless.

If the results are plotted on squared paper as before, the curve obtained is a parabola, showing that the ordinates vary as the squares of the abscissae, or *vice versa*, according to the direction in which the tensions and lengths are set. X

Experiment XVIII

Variation with Diameter

Required.—Three or four steel wires of different diameters, sonometer, etc., as before. An ordinary chemical balance will also be necessary.

Take the thickest wire and stretch it between the spring balance and the opposite pin. The ends must be softened in a flame before they can be wrapped round tightly enough. Set the tension at 12 or 15 lbs., put the movable bridge or slide about half-way along, and measure the length by a beam compass. Bring the idle wire to unison with the other, and preserve it for a standard. Make a mark at each end of the experimental wire with a blunt knife, then release it, and substitute the next thickest wire. Using the same tension as before, move the slide till both wires are in unison, mark the ends, and continue the same process as often as may be necessary. By having both tension and frequency constant, the only quantities varying are diameter and length. The former must be known while the wire is being stretched; for this purpose we cut each wire off at the marks, and weigh it. If we call the length l, the radius r, and the weight W, we have weight = volume × density or $W = \pi r^2 l \times \Delta$. For steel we may reckon $\Delta = 7{\cdot}8$, so that $r = \sqrt{\frac{W}{\pi l \times 7.8}} = {\cdot}202 \sqrt{\frac{W}{l}}$.

A comparison of r and l in several cases should show that they vary inversely as one another, or that their product is a constant.

Example.—

Diameters (calculated).	Length (observed).	Product.
·0151 cm.	39·1	59·04
·0131	44·4	58·16
·0102	57·5	58·65

Experiment XIX

Variation with Density

Required.—Wires of steel, copper, platinum, etc.; sonometer as before.

As in the preceding experiment keep the tension and frequency constant. The products of the length and diameter, which were then constant, now vary inversely as the square roots of the densities. The latter may be found from a table, and compared with the experimental results.

The above relations are explained as follows. According to theory the velocity of propagation of transverse waves in a wire whose tension is τ grammes is $v = \sqrt{\frac{\tau g}{m}}$, where m is the mass in grammes of 1 cm. of the wire, and g is the multiplier necessary to convert grammes into dynes: numerically it is about 981 for our latitude. Now if the radius of a wire be r, and its density Δ, then the weight of 1 cm., which we have called m, is $\pi r^2\Delta$. Substituting, we get $v = \frac{1}{r}\sqrt{\frac{\tau g}{\pi\Delta}}$. But this is not all, for the wire, vibrating in its simplest mode, has a node at each end, hence it is half as long as the wave; and further, the number of waves (n) multiplied by the length of each ($2l$) is equal to the velocity, because a wave-length

is the distance traversed in the time of a complete period (a to-*and*-fro oscillation). We thus have $2nl = v$, and from the previous equation by substitution,

$$n = \frac{1}{2rl}\sqrt{\frac{\tau g}{\pi\Delta}}.$$

We may now read off the following laws (noticing that when a quantity is in the denominator on the right-hand side it varies inversely as n, and so on).

The number of vibrations is—

I. Inversely as the length of the wire.
II. Inversely as the diameter or radius.
III. Directly as the square root of the tension.
IV. Inversely as the square root of the density.

It also varies directly as the square root of the force of gravity, but this is commonly left out of account.

It is worth noting that the velocity of transverse waves is theoretically the same as that which a body would have after falling freely through a height equal to *half* the tension length, *i.e.* the length of wire which by its weight alone would produce the actual tension. For m having the same signification as before, the length of wire weighing 1 gr. is $\frac{1}{m}$ centimetres, so that τ grs. occupy $\frac{\tau}{m}$ centimetres. Now the velocity of a body after falling through a height s is $v = \sqrt{2gs}$. Putting $\frac{\tau}{m}$ for s, we have $v = \sqrt{\frac{\tau g}{m}}$.

A similar expression gives the velocity in air or any other medium, solid or liquid (see Chapter XI.).

The laws just enunciated were published by Father Mersenne (see p. 211) in his *Harmonie Universelle* (Paris, 1631), and are called by his name. As much, though by no means all, of the development of Acoustics is the work of this century, it will be seen that he was one of the earliest pioneers.

Experiment XX

Verification of the Formula $n = \frac{1}{2l}\sqrt{\frac{Tg}{m}}$

Required.—Sonometer, tuning-fork of known frequency, beam compass, spring balance, etc.

Tune a wire to unison with the fork, keeping it stretched by the spring balance, observe both length and tension. Make a mark on each end of the wire with a blunt knife: now release it altogether and cut it off at the marks. Weigh this portion, and divide by its length as measured *before release;* we thus get the quantity m or the mass of 1 cm. Substitute in the formula and calculate the value of n.

Example.—A wire 57·5 cm. long under a tension of 24·0 lbs (= 24 × 454 grammes) was found to be in tune with a certain fork. When cut off, the wire weighed ·720 grs.

Here

$$m = \frac{\cdot 72}{57 \cdot 5} = \cdot 0125,$$

and

$$n = \frac{1}{2 \times 57 \cdot 5} \times \sqrt{\frac{24 \times 454 \times 981}{\cdot 0125}} = 254 \text{ nearly.}$$

The fork was marked 256.

Longitudinal Vibrations

These are of importance in connection with the velocity of sound, and are dealt with in Chapter XI. To excite them, rub a wire in the direction of its length with a resined rag, a wave-motion is then set up resembling that in the wire helix, Expt. XIII., when suddenly pulled and let go, but the note is much higher than in the previous case, and is independent of the tension.

Torsional Vibrations

These are not made use of in music, but are of theoretical interest. Their velocity is given by the formula $v = \sqrt{\frac{R}{\Delta}}$ when R is the coefficient of simple rigidity, or the ratio of shearing stress to shearing strain, and Δ the density (see vol. i. pp. 164, etc.).

Some values of R and Δ for different materials are given below—

	R	Δ
Brass . .	$3 \cdot 44$ to $4 \cdot 03 \times 10^{11}$ dynes	8·38
Steel . .	$8 \cdot 19 \times 10^{11}$,,	7·8
Copper . .	$4 \cdot 4$ to $4 \cdot 47 \times 10^{11}$,,	8·8

We shall see later (p. 127) that the velocity of longitudinal vibrations in a solid is $\sqrt{\frac{E}{\Delta}}$, where E is Young's modulus of elasticity, hence by division, velocity of torsional vibrations : velocity of longitudinal $= \sqrt{R} : \sqrt{E}$. When the values of R and E are substituted this ratio becomes approximately ·6 : 1, or what comes to the same thing, the note emitted by the latter is about a major sixth above the former.

M. Cornu has lately devised a means of making torsional vibrations evident.[1]

The apparatus consists of a sonometer wire, a lantern, a mirror fixed so as to reflect a beam of light at right angles to the wire, and a small light mirror stuck on the wire itself, close to one end. The wire is bowed at the other end, and vibrates transversely as usual, thereby altering the aspect of the mirror, and making a spot of light move sideways on a screen. But it also describes part of a revolution backwards and forwards as well, thus causing an up-and-down motion of the light. Under the

[1] *Comptes Rendus*, vol. cxxi. p. 281 (1895).

two influences a curve is traced out resembling that in Fig. 9.

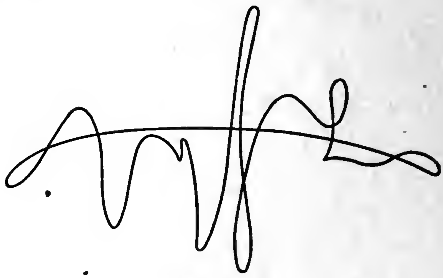

Fig. 9.

The following additional experiments with a sonometer should be performed. A few paper riders, a bow, tuning-fork, and short piece of thin wire are required.

Experiment XXI

Let one of the wires give out any convenient note and gradually bring the other to unison with it. When this is nearly accomplished, observe that either of them, when plucked, will excite the other. At first the ends become shadowy, and a slight tremor can be observed in the middle; but with each successive approximation the vibration is more and more taken up, the amplitude alternately rising to a maximum and falling away to nothing until they are exactly in unison, then they continue vibrating together. This is a very delicate and reliable method, which is independent of the ear. Even when they are precisely in accord, the amplitudes fluctuate a little, because each wire alternately acquires energy at the expense of the other.

The phenomenon of beats (see Chapter XVI.) is also very noticeable here.

Experiment XXII

Draw a bow across the wire at its middle point; no sound except a screech can be elicited. This at first sight appears very remarkable, because when plucked at this point it sounds readily enough: it means that a wire is incapable of giving its fundamental or lowest note when a bow is urging it, unless this note is accompanied by the upper octave; now this requires a node at the middle, which is incompatible with the conditions, for we find that at whatever point a wire is bowed, an antinode forms there (see p. 152).

The observation was first recorded by Delezenne in 1842.

Experiment XXIII

Place three or four paper riders upon the wire at different points and touch it gently with the bow: they all instantly fall off. Lay a feather lightly on the middle, and hang two or three riders on one half while the other half is bowed: again they are thrown off. A nodal point is formed where the feather lies, but as it is not in any way fixed the motion is easily communicated through it.

Damp the wire at $\frac{1}{3}$ of the length from one end, let one rider occupy the $\frac{2}{3}$ position while the others lie anywhere along it, now bow the shorter portion. All of them will be unhorsed except that at the point of trisection. Here three equal vibrating segments are formed, separated by two nodes. Similarly, four or more segments can be obtained, according to the thinness of the wire. Observe that this result depends on the formation of stationary waves, as in Expt. XII. p. 11. Many of the minor waves are present even when the wire is giving its fundamental note (see Chapter XIV.).

EXPERIMENT XXIV

Make a small ring of thin wire in the shape of a figure of 8, let it hang near the middle of the sonometer wire and pluck it: the ring will be observed to rotate first in one direction and then in the other, showing that vibrations of a torsional character are present.

EXPERIMENT XXV

Instantaneous Views of a Vibrating Wire

Required.—Battery or dynamo; induction coil; Leyden jar; sonometer with a bright object, such as a bit of broken glass bead, fastened on one of the wires at the centre; the whole to be in a dark room.

Make the usual electrical connections and place the jar in the secondary circuit so as to make the sparks denser and brighter. Adjust the tension of the wire till it performs a double oscillation in the interval between the sparks: when this happens the bright spot will appear stationary for several seconds, but changes its position by degrees. Now tighten or slacken the wire very slightly, and it will be seen to move slowly from one side to the other, because it is visible at different points of the swing instead of always at the same one. When the frequency of the wire is half that of the sparks, two views of each swing are presented, and so on, and if the amplitude be considerable some curious appearances are observable.

Another variation in the experiment consists in illuminating the wire by flashes of light from a lantern, the interruptions being caused by a revolving wheel with radial slits in it. In this way the rapidity of the flashes is capable of wide variation, and the amount of light much greater, but it is more adapted for lecture purposes.[1]

[1] For further experiments with a sonometer see pp. 135, 145, etc.

CHAPTER IV

RESONANCE

In its wider sense resonance means any augmentation of sound, such as is produced when the shank of a tuning-fork is pressed on a board; the latter then becomes an independent source of sound, and is consequently a resonator. But a more restricted meaning is usually intended, viz. when a volume of air or other gas contained in a cavity vibrates in sympathy with a certain note or range of notes. The effect is easily noticed on bending down over a fairly deep bowl and speaking into it: a position is soon found where the voice, which may have to be altered in pitch, rings out with great power. In a room of whatever size, the air has a particular rate of vibration, which it takes up by preference, and any one singing or speaking on this note is more audible in every part, and needs to exert less effort to be heard, than if his voice is pitched above or below it.

In stringed instruments of music resonance is absolutely necessary to augment their tone. In a piano the wires are stretched over a broad surface of fir-wood; in a harp, violin, 'cello, etc., the resonator is a box with thin walls, and so on. Tuning-forks for acoustical purposes are screwed on to an open rectangular box, whose purpose is to act as a resonator (see p. 64).

The following are some illustrations of the principles under consideration.

Experiment XXVI

Take a deep glass cylinder, such as is used for collecting gases, and let water pour into it from a tap. Amid the noise of splashes it is quite easy to recognise a note which rises in pitch as the unfilled portion gets shorter. The flow may be stopped at a certain point, and the accuracy of the ear tested by trying whether the jar will resound to a fork whose pitch it was judged to attain during the filling.

Experiment XXVII

Hold a tuning-fork to the mouth, adjust the aperture of the lips and size of the cavity till the maximum effect is obtained. Here the mouth is an independent source of sound. Try to use the voice while maintaining everything in its present state: it will be found either that a certain vowel only can be uttered, or perhaps that it is impossible to bring out any note at all.

Experiment XXVIII

Required.—Cylindrical glass jar; piece of card; several tuning-forks; ether.

Sound a fork and hold it over the empty jar (see Fig. 11): probably there will be little or no change in the intensity. Pour in a little water and test it again: if an augmentation is noticed, keep on adding water till the effect is a maximum. It may be that the jar was too short originally, in that case the aperture must be shaded with a card. Now use another fork: it will be found that a fresh adjustment is necessary. Pour in a few drops of ether:

the resonance is destroyed; but on adding a little water it is restored once more.

Blow across the aperture of the jar: if not too wide it gives a note nearly the same as that which it reinforces.

Hold an empty jar close to the ear, it seems full of sound, every footstep or clatter excites a responsive echo in it, and if its own note be struck forcibly on a sonometer or piano, the effect on the ear is somewhat violent. The other ear should be stopped with a finger meanwhile.

A jar once tuned to a note may be kept constantly ready by pouring in melted paraffin to the proper depth instead of water.

Experiment XXIX

To measure the Length of a Resonating Column

Required.—Apparatus shown in Fig. 10. It consists of a wooden base and standard, the latter supporting a tube T, about 20 cm. by $2\frac{1}{2}$ or 3 cm., and open at both ends. This tube can be lowered to any required depth into a somewhat wider cylinder C, which is nearly full of water, thereby converting it into a stopped pipe of variable length. T′ is another tube of smaller dimensions to be used instead of T, and several tuning-forks of known frequency should be provided.

Strike one of the forks, hold it as shown at F, and adjust the level of water in T till the greatest reinforcement is obtained; measure the distance from the top to the water level, add on ·8 of the radius (because the effective length is to this extent greater than the actual one), take the mean of a number of such measures and call it l. We thus get the distance between a node and an antinode, or $\frac{1}{4}$ of a wave-length. This is connected with the velocity of sound by the relation $v = 4nl$, where n is the frequency. Compare the result with that obtained by dividing n into v, which is assumed to be known.

The following is a specimen of a result :—

Frequency 288, length of resonating column (4 observations) 28·3, 28·6, 28·2, 28·6 ; mean = 28·47 cm.

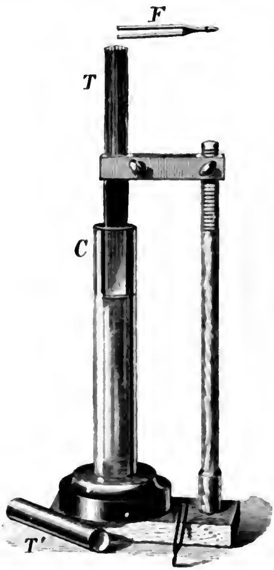

FIG. 10.

Diameter of tube 2·2 cm., correction (·8r) ·88 cm. Adding this, the effective length is 29·35 cm.

Velocity of sound 33,000 cm. per second; temperature in tube 14·3° C.; correction for 1° = 60 cm. per second. Hence velocity at 14·3° = 33000 + 60 × 14·3 = 33858.

Dividing by the frequency (288) gives for the wave-length 117·56, but this is four times that of the pipe, as explained above, so that the latter should be 29·39, whereas it was found to be 29·35.

The error then is ·04 cm.

To make a refined experiment the effect of humidity in the tube, making the air lighter and thus increasing the wave-length, must be allowed for. The rate of the fork must be checked by comparison with a pendulum (see p. 76), and a correction applied for the temperature, equal to about ·00011 of a vibration for each degree C., to be subtracted for an increase and added for a decrease, reckoning from its temperature while being rated. A very minute amount of flattening also takes place owing to the open end being shaded while the fork is held over it, but this may be neglected (see also p. 132). The sources of error are, first, that it is at all times difficult to determine a maximum; far better results can be obtained in

"null" methods, *i.e.* when some effect is reduced to zero, but these are for the most part impracticable in acoustics; and secondly, the correction ·8r is only an approximation; it probably has a special value for every diameter of tube, and according to some determinations, it should be ·6r and not ·8r.

Experiment XXX

Procure an open tube of the same diameter as the one just employed, and nearly twice as long. Provide it with a slide made of stout drawing-paper, so that it can be lengthened if necessary. Hold the fork in position, and measure the length which gives the maximum resonance. In theory it should be twice as long as before, because it is now equivalent to two such tubes placed end to end. In practice it is rather more, the precise conditions assumed by theory not being fulfilled.

Principle of the Resonator.—Let AB, Fig. 11, be a cylinder containing a wire helix fixed at B; press the topmost coil with the finger, and wait for the natural period of recovery of the wire, then time the impulses so as to augment the motion. If this be not done there are frequent interruptions to the regularity of vibration. Now remove the helix and place a tuning-fork at the mouth instead. Then if the length be properly chosen, the air becomes alternately condensed and rarefied in much the same way, and it is easy to see that were the jar too short or too long, the air

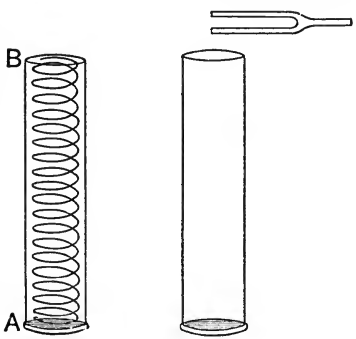

Fig. 11.

pulses would arrive too soon or too late for the fork, and they would conflict with one another.

When the two are in accord, however, a much larger volume of air than the unaided prongs could affect is set into sympathetic vibration, hence the increase in loudness. The energy necessary for this is taken from the fork, which therefore comes to rest sooner than if free.

The whole effect, though involved in what has been said, needs further elucidating. We have seen that from the fixed end of a coil, and the same is true of a column of air, a condensation or a rarefaction is reflected without change of type, while from the open end there is such a change. Before a complete cycle has elapsed then, *i.e.* before a compression travelling downwards is followed by the next compression in the same direction, the four following changes have to be executed :—I. The compression reaches the bottom. II. It is reflected upwards as a compression. III. From the upper open end it returns as a rarefaction. IV. This rarefaction is reflected as such from the bottom. Lastly, it is sent down as a compression and the cycle is repeated. No. I. coincides with the downward swing of the fork, No. III. with the upward, and so on. Hence the length has to be traversed four times in the time of a complete period (a to-and-fro vibration) of the fork.

In a tube open at both ends a compression at one end is returned as a rarefaction from the other; this at the next reflection becomes a condensation again, and so on. Hence the length of the pipe has only to be traversed twice, but to compensate for this it is twice as long.

It is hardly necessary to remark that, although a very long tube does not act as a resonator, it *conveys* sound well enough, as in a speaking-tube; reflections take place from each end in the usual way, but they are extinguished by friction against the walls, and by meeting impulses of superior power.

For acoustical purposes, resonators of thin brass, spherical or cylindrical, are employed (Fig. 12). They are provided with two holes, the wider to catch the sound, the other to lead it to the ear. The diameters vary between $15\frac{1}{2}$ cm. and $4\frac{1}{3}$ cm.; smaller ones are of no use. Such resonators have a very small range, and are employed to pick out a note from a composite sound, by reinforcing it at the expense of the others. The walls, if

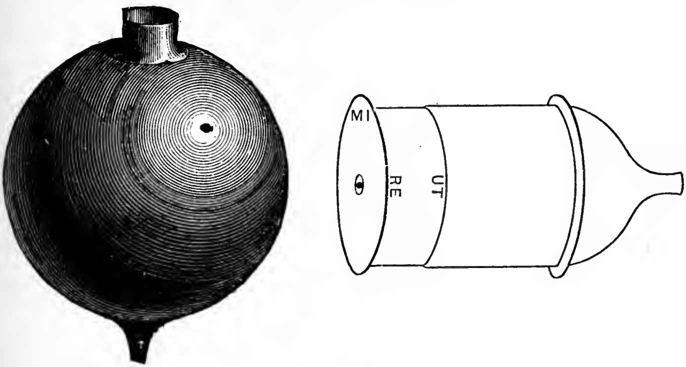

Fig. 12.

thin, are thrown into vibration, thus enabling a resonance to be felt as it were. By shading the aperture slightly, the proper note of a resonator is lowered, because the inertia of the external air has less effect upon it; if a slight sharpening is required, put in a little melted wax, and rotate it so that it lies uniformly inside. In order to use a resonator effectively no sound must reach the ear except through the instrument itself: for this a short rubber tube, with a nipple at the end, may be employed. The other ear must be closed entirely. The shape of the enclosed air, so long as its volume remains the same, has little influence, for if a glass flask be tuned to a fork

by pouring water in, it may be tilted without altering the result. Nor does the shape of the aperture make any difference if its area is unaltered.

The smaller hole must in all cases be stopped, or the resonance is very much weakened. When applied to the ear, the drumskin acts as a stop. A cardboard resonator may be made without much difficulty, shaped as in Fig. 13.

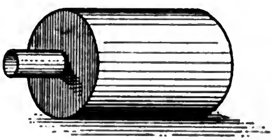

FIG. 13.

The dimensions within wide limits are immaterial, as the size of the aperture is a controlling factor, and this can be varied afterwards. A short tube may be fastened on as shown, the effect being to make the pitch flatter, while enlarging the hole makes it sharper. The cylinder is conveniently made in two parts, one of which slides into the other like the draw-tube of a telescope; it can then be used for several notes, while the aperture is left constant.

EXPERIMENT XXXI

To detect Resonance by a Mechanical Effect

Required. — Resonator, and fork to correspond; thin copper foil; fine thread; wax.

Place the resonator with its aperture in a vertical plane, and close up the smaller end. Cut out a circular or rectangular piece of foil somewhat less in area than the aperture: hang it by a thread just in front, and at right angles to it. Now strike the fork: the disc will immediately set itself in the plane of the aperture, so as to close it as far as possible. This is because the mean pressure inside is greater than it is outside, and a slight current is set up, causing the disc to place itself in the manner indicated. Similarly, when a coin is thrown into water it falls in a horizontal position, though

not without swaying from side to side. The method described may be made of any degree of sensitiveness by using a small silvered glass disc, from which a beam of light is reflected on to a scale. (See a paper by Professor Vernon Boys, *Nature*, vol. xlii. p. 604, 1890.)

Experiment XXXII

The Phoneidoscope

Required.—Glass cylinder; tuning fork; soap solution.

Make the cylinder resound to the fork as usual: draw a glass rod, previously dipped in the solution, over its mouth so as to stretch a film over it. Now, on striking the fork, and looking at the film by reflection, it is seen covered with wrinkles: the play of colours superadded makes the effect very beautiful. The experiment in its most improved form is due to Mr. Sedley Taylor,[1] though it had been suggested by Helmholtz.

Experiment XXXIII

Acoustic Repulsion and Attraction

Required.—Resonator and fork of the same pitch, wax candle, narrow strip of wood, pin, and counterweight.

Set up the apparatus as in Fig. 14. The candle is inverted and fixed in an upright position, a pin is pressed into it while warm, and held till it is set. The wooden strip is bent while being held over a flame, and keeps its shape on cooling. Fasten the resonator to one end, and put a counterpoise on the other: balance the whole arrangement as shown. Now, on holding a fork in the proper position, a rotation is set up in the direction of the arrow. It is due to the reaction of the current which issues from

[1] *Proc. Roy. Soc.* 1878.

the aperture as in Expt. XXXI. When the fork is held close to the opposite end, or even to a suspended rectangle of paper, there is attraction instead of repulsion. The mathematical theory of both effects is given in Lord

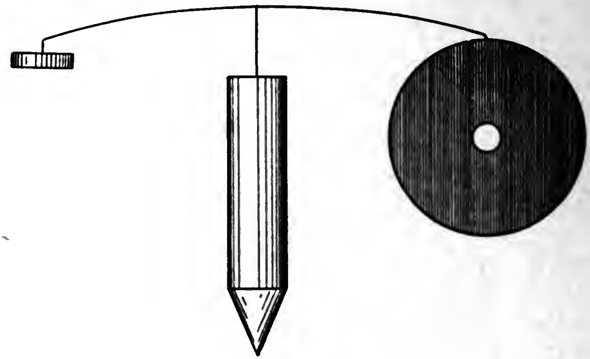

FIG. 14.

Rayleigh's *Theory of Sound*, vol. ii. p. 42. They were noticed almost simultaneously by Professors Mayer of Hoboken and Dvořak of Agram. An acoustic *reaction-mill* is sometimes used, identical in principle with the above, but containing four resonators suspended to a cross-piece, and balanced on a steel point.[1]

EXPERIMENT XXXIV

The Acoustic Pendulum

Required.—Small brass ring, such as is used for a reading-glass; tissue paper or goldbeaters' skin; small ball of wax hung by silk thread.

Stretch a sheet of tissue paper or goldbeaters' skin over

[1] For Bjerknes' experiments on the attraction and repulsion of vibrating bodies, see *Nature*, 8th June 1882.

the ring, and from the rim hang a small ball of wax by a fine thread so that it reaches to the middle. This instrument is called an acoustic pendulum. When provided with a resonating cup its efficacy is increased, the ball being constantly knocked about with greater or less degrees of force. It can be used for exploring the wave systems between a source of sound and a vertical wall:[1] it is quiescent at a node, but agitated at an antinode. The ear, on the other hand, which only receives impressions from *one* side, hears nothing at an antinode, but does at a node.

A body whose vibrations are soon damped will resound to almost any note, but when they are prolonged for a considerable time, as is the case with a tuning-fork, the least deviation destroys the effect.

[1] This was done by N. Savart, who used it in conjunction with a large organ pipe to find the positions of the nodes and antinodes (see p. 165).

CHAPTER V

DETERMINATION OF FREQUENCY

EXPERIMENT XXXV (METHOD II)

ONE method has already been described (see p. 28).

Required.—Tuning-fork, whose rate is to be measured; rectangular steel strip, about 30 cm. by 2 cm., thin enough to be readily flexible; vice.

Clamp the strip vertically in the vice, and set it vibrating. Adjust the length till it makes 5 double vibrations in a second. This can be done with fair accuracy by comparison with a clock beating seconds, or even a watch. Measure the length (L) of the vibrating portion. Now shorten it till it gives the same note as the fork; call this length l. Then, the ratio of vibration being inversely as the squares of the lengths, we have n (the frequency required) $= 5 \times \frac{L^2}{l^2}$. No great accuracy can be expected from this method, but the mean of several trials will give a fair approximation. It was employed by Chladni in the construction of a tonometer.

Example.—

Length of strip vibrating 5 times per second, 16·22 cm.
,, ,, ,, in unison with fork, 3·70 ,,

$$n = \frac{5 \times 16 \cdot 22^2}{3 \cdot 7^2} = 251 \cdot 1 \text{ instead of } 256.$$

EXPERIMENT XXXVI (METHOD III)

Required.—Savart's wheel.

This instrument possesses historic interest, but is now little used, except for qualitative experiments. A toothed wheel is made to revolve at any required speed by a multiplying arrangement, and a card is held against the teeth. The taps are heard separately when it revolves slowly, but blend into a continuous note as the speed increases. To make a determination, the rate must be run up till the note is the same as that of a fork or organ pipe, and maintained at this point for a given time. Then the number of teeth in the wheel multiplied by the number of rotations, and divided by the time in seconds, will give the required frequency. The sources of error are numerous, and far more accurate determinations can be made in other ways.

EXPERIMENT XXXVII (METHOD IV)

Required.—Sonometer.

Make one wire give out the note whose frequency is to be determined, and measure its length. Cause the other wire to vibrate 5 times per second faster; this can be done with great accuracy by tightening it slightly after unison is reached, and causing the beats to attain this rate (5 per second). Now shorten the first wire till it is exactly in tune with the other (leaving the tension unaltered), and measure it again. Let l_1 and l_2 be the lengths, then the frequencies are n and $n+5$ respectively. By Mersenne's first law, $\frac{n}{n+5}=\frac{l_2}{l_1}$, from which $n=\frac{5l_2}{l_1-l_2}$.

The theory of this method is unobjectionable, but in practice the lengths l_1 and l_2 cannot be found with sufficient accuracy to make it reliable.

EXPERIMENT XXXVIII (METHOD V)

Required.—Siren and bellows; chronometer or stop-watch; tuning-fork; bow.

Description of the Instrument.—A perforated metal disc, about 6 cm. diameter (Figs. 15 and 16), revolves on a vertical axis between two supports. It is pierced with a

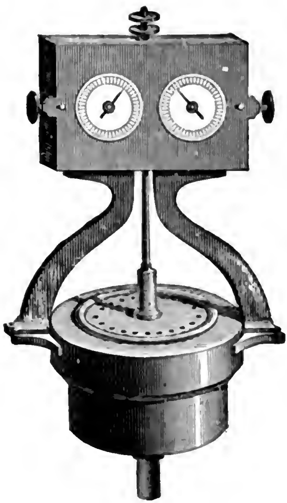

FIG. 15.

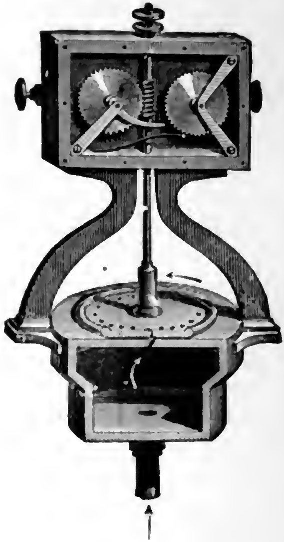

FIG. 16.

number of holes (20 in the figure), which are equally spaced, not parallel to one another, but equally oblique to the axis; underneath it lies the cover of the wind-chest (cut open in Fig. 16), in which there are the same number of holes, slanting the opposite way. Air is forced through from a bellows, and can only escape when the two sets of holes

are opposite one another. The reaction due to the slanting sets up a rotation as shown by the arrow: consequently the air issues in puffs at a rate depending on the number of turns per second. This is registered by a counting gear, consisting of a worm and worm-wheel which can be thrown into or out of action by pressing a button. The right-hand dial (Fig. 15) registers single revolutions, the left one hundreds of revolutions.

To make a determination, two observers are necessary, one to work the bellows and attend to the tuning, the other to set the counting gear in action and take the time.

It is of great advantage to have a sustained sound for comparison, such as a singing flame (see p. 178) brought to unison with the note to be tested. A tuning-fork may also be employed, but requires exciting at intervals.

Having arranged that both needles point to zero, draw the worm-wheel aside so that it no longer gears with the worm, and work the bellows. At first a succession of puffs is heard, but these soon blend into a continuous sound, which gets louder as the rate is increased. As it approaches the note under examination, the usual beats are heard, and probably it would give at least as accurate a result if they were attended to and kept constant instead of the unison. It is more usual, however, to bring the notes together, the beats then disappearing entirely, and at a suitable moment the counting gear is started and maintained in action for a minute or some such interval, then thrown out again. Say the dials register 852 revolutions in 60 seconds, and that there are 20 holes in the disc, then the number of puffs per second is $\frac{852 \times 20}{60} = 284$. If there were only one hole in the lower disc, it would not affect the pitch, but it would make the intensity very much less.

The determination is liable to several sources of error.

The noise made by the bellows and the effort of working them distract the mind somewhat, so that it is difficult to maintain a unison during the whole interval. This difficulty is aggravated by the tendency of the disc to accelerate, so that if a constant pressure of air is maintained, it does not give a constant note. In an improved form of the instrument, first used by Helmholtz, and now procurable, the holes in both discs are upright, and the rotation is kept up by a motor; in this way a very accurate determination may be made.

In practice, only tuning-forks require to have their frequencies known very exactly, notes on other instruments being determined by comparison with these.

Helmholtz Siren (Fig. 17).—This instrument may be described here, but its principal use is rather to illustrate some of the principles of harmony, interference, etc., than for determining frequency.

It contains two revolving discs, one at each end of the axle (only the lower is exposed in the figure); both are pierced with four rows of holes, viz. 8, 10, 12, 18 in the lower, 9, 12, 15, 16 in the upper. Wind is admitted through the large supply-tubes, but all or any of the rows of holes may be opened or shut at pleasure by pins. The boxes shown at the side are simply hollow cylinders of brass intended to reinforce the prime tone at the expense of the upper partials. On the middle of the axis a screw thread is turned for the purpose of working the counting gear. The revolving discs move immediately over or under a fixed disc as in the ordinary instrument, but before reaching either of these the air has to pass through another set of holes contained in flat rings, of which there are eight in all (four to each disc), any one of which can be moved through a small angle by pushing the pin in connection with it. The first pin on the right, whether above or below, controls the outermost ring, as will be evident from

the figure, and the fourth one the innermost. In its ordinary position the holes in any ring do not coincide with those in the fixed disc, so that no sound is produced till at least one pin is pushed in, but when this is the case the air finds no obstacle till it meets the movable disc,

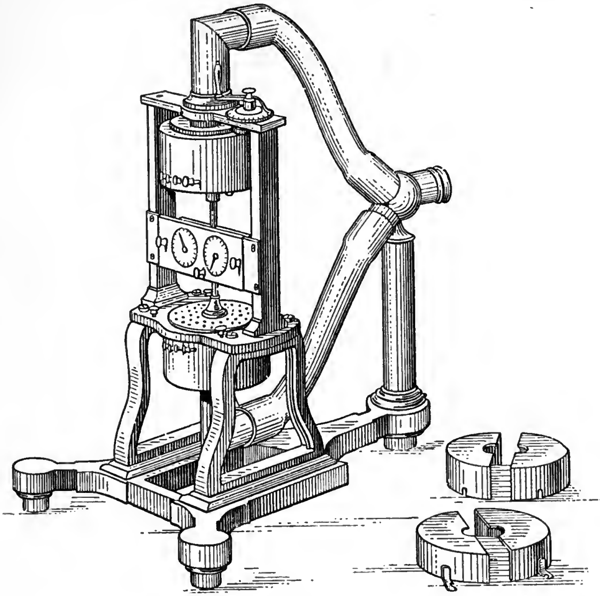

FIG. 17.

then it issues in puffs as usual. A handle shown above the cross-bar turns a cog-wheel gearing with another one on the top of the upper box, and causes what has hitherto been called a "fixed" disc to revolve with one-third of the speed of the handle as measured by the pointer. By this means additional powers are conferred upon the instrument. Suppose we wish the puffs of the lower disc to alternate

with those on the upper, we push in the 2nd pin below and the 3rd above, thus setting the common row of 12 holes in action, and turn the handle 45°; this, owing to the difference in size of the cog-wheels, moves the "fixed" disc through $\frac{1}{3}$ of that amount, or 15°, so that each hole is now $\frac{1}{24}$th of a circumference from where it was. The effect of this on the sound is to quench the fundamental or prime tone, but to strengthen the upper octave and all the higher tones which have 4, 6, 8, etc., times the frequency of the fundamental.

Lastly, by continuously revolving the handle, the puffs from the upper instrument can be increased or diminished in rapidity, as compared with the lower, and the intervals thrown slightly out of tune.

EXPERIMENT XXXIX (METHOD VI)

Required.—Appun's Reed Tonometer.

This consists of a long rectangular box with a "foot," by which it can be set horizontally in a hole on a wind-chest. It contains 65 reeds, each placed over a slit, as in a harmonium, giving an octave from 256 to 512 vibrations per second. An equal number of buttons outside enable any one or more of them to be sounded. Each reed performs 4 vibrations per second less than the one on its right. In order to find the frequency of any note within the compass of the instrument it is only necessary to see which reed is in unison with it, or, failing that, the one which is nearest, and add or subtract the number of beats per second, according as the reed is the lower or the higher of the two. The only objection to this method, which is otherwise extremely convenient, is that the elasticity and rate of vibration of a reed alters slightly with the temperature, becoming less for a rise and *vice versa*, so that it can only be depended on at the temperature when it was originally tuned. When two reeds are sound-

ing together they have a tendency to influence one another, but for an experiment such as we have described this is not the case.

A far more costly instrument than the above is Scheibler's tonometer, in which the reeds are replaced by tuning-forks. There are 65 in all, covering the same range as before. As prepared by Dr. Koenig two others are added, giving F and A in the same octave. The method of use is the same as that indicated above.

Dr. Koenig has prepared a still larger series ranging from 16 to 21845·3 vibrations per second: there are in all 154 forks, most of which are provided with adjustable resonators, and also with sliding weights by which the rate can be varied within certain limits.

CHAPTER VI

VIBRATIONS OF RODS, TUBES, AND PLATES

THE motions of tubes are the same as those of rods, but the lightness of the former makes them more convenient for some purposes.

Either of them may vibrate either longitudinally or transversely, and they may be fixed at one or both ends, or free at both ends and supported at some intermediate point or points.

When fixed at both ends the longitudinal vibrations are like those of a stretched string: the transverse ones (since they depend on elasticity, not tension) obey a different law. For example, the vibrations of a string are inversely as the length: when damped in the middle it has twice the frequency; at a point of trisection three times, and so on. But a rod fixed at both ends and giving its fundamental note has a frequency which, to avoid fractions, we may call 9 or 3^2; with one node in the middle it becomes 5^2, with two nodes 7^2, and so on.

When fixed at one end its (transverse) frequency is denoted by the formula $n = \frac{c\theta}{l^2}\sqrt{\frac{R}{\Delta}}$, where θ is the thickness, R the modulus of rigidity, and Δ has its usual signification of density, while c is a constant whose value is about ·28. It will be observed that the breadth is not a factor.

A rod free at both ends can vibrate with two, three, or more nodes, but not with one. The formula as given by

Poisson is $n = \frac{v \times 1 \cdot 0279\,\theta}{l^2}$, where v is the velocity of longitudinal vibration and θ the thickness. Its fundamental is higher than if fixed at one end in the ratio of 25 to 4, and its notes of subdivision follow the squares of the odd numbers (see below).

The longitudinal vibrations of rods and strings are of great interest in connection with the velocity of sound, and are dealt with in Chapter XI.

Experiment XL

Transverse Vibrations of a Rod fixed at one end

Required.—Stout brass wire about $1\frac{1}{2}$ m. in length; vice.

Make the wire as straight as possible, clamp it at the lower end and pull it aside, to set it swinging throughout its whole length, as in Fig. 18, I. Now hold it about $\frac{1}{3}$ of the way from the top and shake the remaining portion in the middle, and it will vibrate as in II; and similarly, the state shown in III can be induced.

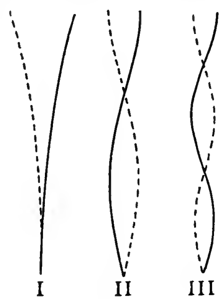

Fig. 18.

The frequencies depend in the first instance on the distance from the free end to the first node: these are nearly as $1 : \frac{1}{3} : \frac{1}{5}$, and the corresponding rates are inversely as the squares of these numbers; that is to say, while No. III executes twenty-five vibrations, II will execute nine, and I only one.

Experiment XLI

Transverse Vibrations of Rods free at both ends

Required.—Several glass rods or tubes about $\frac{1}{2}$ cm. in diameter, and of various lengths up to 30 cm.; file.

Hold these tubes a short distance above the table and let them drop one after another: the clinks are sufficiently musical to suggest certain intervals, and by suitably altering the lengths they may be tuned to give the notes of a common chord. Support all the four tubes at their nodal points (about $\frac{1}{4}$ of the length from the ends) and tap them with a pencil: each will give the same note as before. The simplest way is to tie them to two pieces of string, knotted at the proper points, and to let them hang in a vertical plane, while the tubes are horizontal. The instrument known as a Xylophone is merely an amplified arrangement of the same kind, with bars of wood instead of glass tubes. Tuning-forks are founded on the same principle, but are of sufficient importance to be dealt with separately.

EXPERIMENT XLII

Vibrations of Plates

Required.—Chladni's plates, square and round; bow; fine sand; lycopodium. It is advisable to have a clamp which has a sufficient span to reach to the middle, so that a plate may be held either there or elsewhere (Fig. 19).

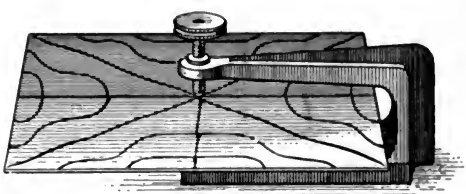

FIG. 19.

In default of this, an inverted iron triangle with the feet covered by rubber, or cork, may be used.

Taking first a square plate, clamp it in the middle, and

draw a bow down one of the sides half-way along: this will excite the gravest possible note. Hold the ear just above the plate at different points: notice that over the diagonals there is hardly any sound, while elsewhere it is very loud. This is a case of *interference*, to be more fully described in Chapter XVI. Draw the bow along the edge at various points and press a pencil on the plate at different places to damp the motion there: it will be found that the note undergoes many variations, but sometimes it is impossible to elicit one at all until a slight adjustment is made.

Experiment XLIII

Sprinkle fine sand over the plate, covering it uniformly, but very thinly; excite with the bow as before and observe the pattern. Repeat the experiment, damping the plate at various places with a pencil: each note has its own pattern. Much of the variety is lost when the plate is always clamped at its centre: clamp or support it, therefore, at different points. It is obvious that the sand seeks the lines where there is least motion, and on either side the adjoining sections are always moving in opposite directions, one up and the other down. Pressure with a blunt point anywhere determines the passage of a nodal line through that point.

It was long ago remarked by Strehlke[1] that the sand lines are not straight, but hyperbolic, and that they avoid crossing one another as far as possible.

Experiment XLIV

Use a circular plate clamped in the middle, and observe that it gives radial lines more easily than any others, but

[1] *Phil. Mag.* 1831.

by very oblique bowing it is possible to obtain a circular one, with 10 or 12 radii.

Here again, the improvement caused by clamping the plate at other points than the centre is very marked. If there be a hole through the centre various combinations of circles and radii can be obtained by drawing a resined string up and down it.

While the plate is in motion press a pencil on one of the radial lines and move it round slowly: all these lines then move in the same direction, and can be kept oscillating backwards and forwards for some little time.

Experiment XLV

Now sprinkle lycopodium or silica over the plate, instead of sand. These powders avoid the nodal lines, and congregate in little heaps around the places where the agitation is greatest. This phenomenon was observed by Chladni, but first explained by Faraday, who ascribed it to the formation of miniature cyclones in which the particles are kept suspended, being too light to force their way through. When in a vacuum all powders, light or heavy, seek the nodal lines. This experiment is seldom performed, but can be done by soldering a brass rod to the middle of a thin metal plate, and passing it up through a stopper in the tubulure of an air-pump receiver (Fig. 20). The rod then being excited longitudinally, the plate vibrates transversely, and any powder on it moves as described.

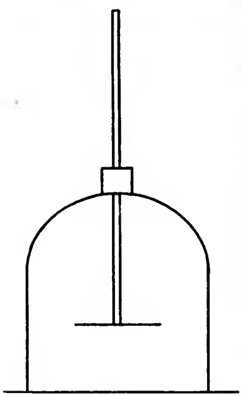

Fig. 20.

Experiment XLVI

Clamp a circular plate in the middle, scatter lycopodium on it, and draw the bow obliquely downwards: it is possible then to make the powder lie in a circle, and to travel all round it; *i.e.* every particle running over the whole circumference. The motion here is akin to that of a wine-glass when a wetted finger is drawn round the rim.

Sand put on the plate when in this state is violently agitated, but gives no definite pattern: as the motion dies away it may be observed to throb at intervals while the wave is passing under it.

Hold a resonating glass flask over the heaps of lycopodium, they are blown about in a circle. A small disc held over makes them contract (see p. 41).

Experiment XLVII

Make a circular disc of cardboard of the same diameter as the plate, and cut out three or four alternate sectors corresponding with those shown by the sand lines (which of course must be arranged to suit the case). Place this disc immediately over the metal plate and turn it round. The sound will be observed to rise and fall according as the open spaces are over the vibrating sectors, or half over one and half over the next. This is again an effect of interference, much the same in principle as that observed when one prong of a tuning-fork is shaded, and the other left free (p. 68).

In theory the smallest number of vibrations of a plate, whether round or square, is given by the formula $n=\frac{c\theta}{A}\sqrt{\frac{E}{\Delta}}$, where θ is the thickness, A the area, and c a constant, while E and Δ have their usual significations. In other words, it is directly as the thickness and inversely

as the area, so long as the material remains the same. This formula, substituting A for l^2, is the same as that which gives the frequency of a rod of length l, clamped at one end.

Wheatstone's Theory of the Sand-figures on a Vibrating Plate[1]

Fig. 21, I, shows a metal strip free at both ends; its first three successive modes of vibration are as in II, III, IV; one additional nodal point appearing in each case.[2]

Now suppose the length and breadth of the strip to be equal, so that it becomes a square; it can then vibrate as

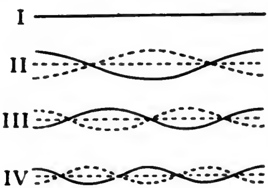

FIG. 21.

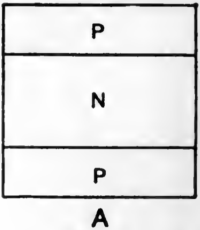

FIG. 22.

shown in Fig. 22, with two nodal lines across it. We may distinguish the three rectangles thus formed by the letters P N P, denoting that the external ones are above the general level, while the medial one is below it.

Since the plate is supposed perfectly symmetrical, it may also (at the same time) divide as in Fig. 23, B and C, and not only so, but the superpositions of the motions

[1] *Phil. Trans.* 1833.

[2] The distance from a free end to a nodal line is rather less than half the distance between two adjacent lines, and the frequencies are as $3^2 : 5^2 : 7^2$. These conclusions were established by Euler.

may be different; that is, either B or C may be superposed on A.

Now draw the combined figures, representing P by horizontal shading, and N by vertical. This will give us either D or E in Fig. 23, according as B is on A, or C on A, respectively. It is obvious that since the motions in the

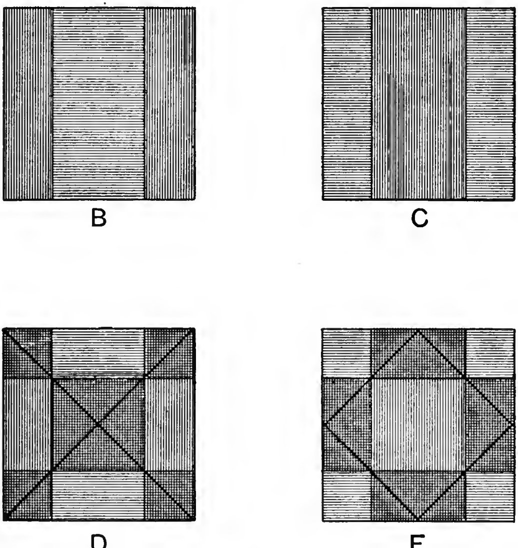

FIG. 23.

rectangular segments in D and the square ones in E are in the same direction, there is no conflict between them, and no nodal lines are formed there, but in the other segments there is a conflict, and considerations of symmetry demand that a couple of diagonals or a square will result, as shown.

Wheatstone lays down the rules as follows:—"(1) The points where the quiescent (nodal) lines of one figure intersect each other remain quiescent points in the resulting

figure. (2) The quiescent lines of one figure are obliterated when superposed by the vibrating parts of the other. (3) New quiescent points, which may be called points of compensation, are formed whenever the vibrations in opposite directions neutralise one another. (4) At all other points the motion is as the sum of the concurring or the difference of the opposing vibrations."

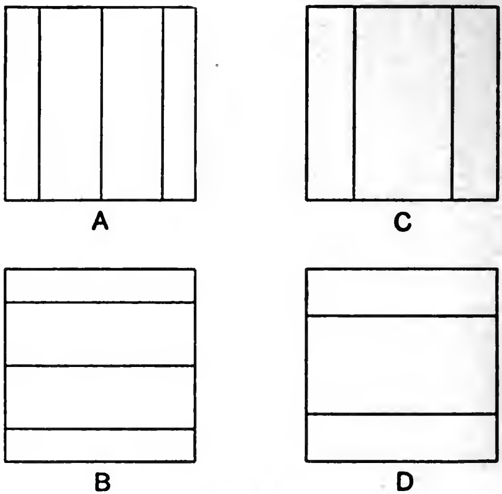

FIG. 24.

Take now a more complicated case where the four figures A, B, C, D, which are in symmetrical pairs, one having two nodal lines parallel to a side of the square, and the other three such lines, are combined. Draw the figure resulting from the combination of all four, as in Fig. 25. Without following out each separate motion, it will be sufficient to notice that the whole square is divided into 36 rectangles, the crossed shading lines making a kind of chessboard pattern. The resulting nodal lines are

then traced as follows. When a crossed segment appears at a corner of the plate, a diagonal line goes through that corner, and on through adjoining sections: when one forms elsewhere along the edge of the plate, the nodal line starts from the middle, and is continued diagonally through the adjacent crossed segments as far as possible. Thus we get the figure shown by the heavy lines. Similarly, the sand lines of any other combination may be predicted.

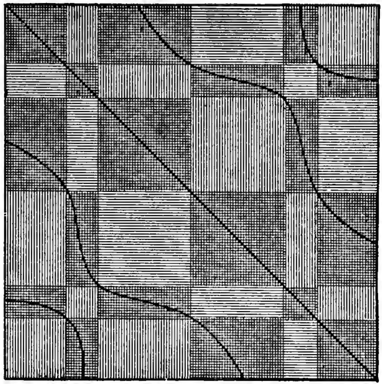

FIG. 25.

It is not necessary, however, that the original nodal lines should be parallel to an edge of the plate: they may take intermediate positions as in Fig. 26, I. One

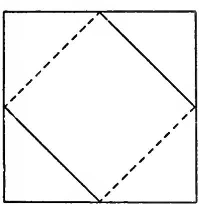

FIG. 26, I.

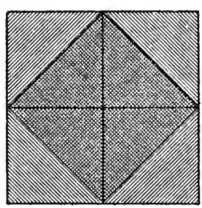

FIG. 26, II.

pair is shown by full, the other by dotted lines. When these are superposed, we have an interior square of crossed shading, which divides diagonally as shown by the thick lines (Fig. 26, II.). The other possible mode, where the

external triangles are filled up with crossed lines, leads to no result.

Take again the primary lines in A (Fig. 27), to which of

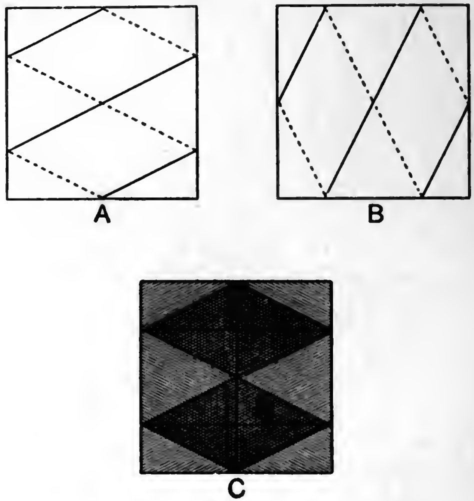

Fig. 27.

course there is a symmetrical but differently disposed

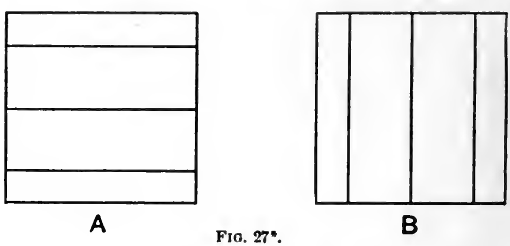

Fig. 27*.

figure B: the four motions may be compounded as in C,

with the nodal lines as indicated. Any other combination may be treated in the same manner.

The assumptions of Wheatstone considered above are not theoretically justifiable when applied to plates, and the figures are ideals, nevertheless they are worth consideration, and the process always gives intelligible results.

Exercise.—Find the sand-figure for a plate divided by three nodal lines parallel to one side and three parallel to the adjacent side (A and B, Fig. 27*).

CHAPTER VII

TUNING-FORKS

No acoustical instrument is of more value than the tuning-fork, it can be made to give any desired note, and the only variation it is subject to is a very slight one for temperature, equal to about $\frac{1}{9000}$ of a double vibration for 1° C., and due to an alteration of elasticity, diminishing when it is warmed, and *vice versa*.

It consists of a steel bar, bent so that the prongs are parallel or even slightly convergent, and having a stem or shank welded on at the bend, which serves as a handle, or as a means of screwing it on a resonance box. It is excited either by striking it on a padded surface, or by a bow, or when the prongs converge, by drawing a wooden rod forcibly between them. In some instruments a pair of sliding weights are provided, which can be clamped at different points, thus altering the pitch within certain limits.

The box is open at one end, and since the aperture is wide, it has a considerable range of sympathetic vibration. If it had just the proper note of the fork, and a very narrow range, there would be an extremely loud sound at first, but the absorption of energy would be so great that it would very soon cease.

The mode of vibration is founded on that of a straight bar, which in its simplest form has, as we have seen, two nodal lines, at nearly $\frac{1}{4}$ of the length from each end

(Fig. 28, I). When bent, a wave-motion has more difficulty than before in getting round the curve, consequently the lines tend to approach one another, as in II. In III the same tendency is accentuated, and in IV they are almost together. The prongs are now more like pendulums, and their period is greater than before on account of the increase in length. The shank moves up and down, and is thus able to set a board in vibration.

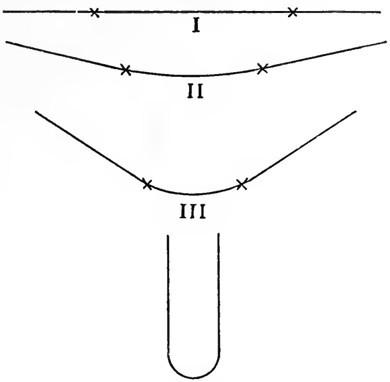

Fig. 28.

The chief drawback to a tuning-fork is that its vibration is not sustained, and it is a matter of some difficulty to keep it so. A crude method is to attach a fine metal point to one of the prongs, so that just beyond the middle of an outward swing it dips into a mercury cup, with a shallow layer of alcohol at the top to keep it clean. A small electro-magnet is placed in the same circuit as the fork, and attracts the upper prong whenever the current passes. The vibration is started by a bow, and maintains itself by the alternate attraction and release when the current is made or broken. A very firm attachment for the fork is needed, or it soon begins to work loose. This method is open to the objection that the impulse is exerted at the wrong time. In order to maintain a free vibration it must act when the fork, pendulum, or whatever it is, is moving with its greatest velocity, *i.e.* at the middle of the swing: if this be not done, the vibration is more or less

forced. Some improvement follows when the magnet is made so short as to go between the prongs, and it is found better not to use a mercury cup, but an ordinary platinum contact-breaker touching a boss on one of the prongs, and this not at the extremity, but some distance away (see Fig. 29). More elaborate methods have been suggested where the chief difficulty is at least minimised, though not abolished.[1]

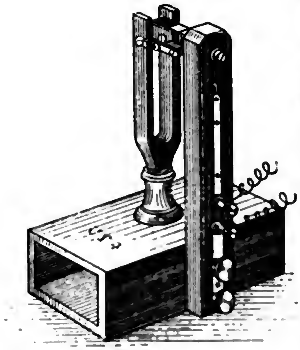

FIG. 29.

A fork may be raised or lowered in pitch by filing the ends or the curved part between the prongs respectively. For ordinary purposes, when two forks are to be used in conjunction, the one which is too high in pitch is weighted with soft wax.

The following experiments should be tried. A single small fork is sufficient for most of them.

EXPERIMENT XLVIII

Notice the extreme ease with which a fork is set into vibration, a single flick with a piece of thread, or a sharp puff of air from the mouth, causing it to ring audibly.

EXPERIMENT XLIX

Tap it somewhat forcibly on the knee and observe the shadowy appearance of the prongs. Move it rapidly back-

[1] See a paper by Professor S. P. Thompson, *Nature*, 22nd July 1886; also W. G. Gregory, "On Driving Forks Electro-magnetically," *Phil. Mag.* Dec. 1889.

wards and forwards in the plane of vibration: the prongs now appear multiple, because their oscillation being alternately with and against that of the hand, they are stationary at certain points. If a bright object be stuck on the side of a prong, the experiment succeeds rather better.

Experiment L

Fasten a fine needle or piece of wire to a prong by soft wax, so as to form a continuation of it. Smoke a piece of paper or glass by holding it over a flame of turpentine: draw the needle along it while the fork is vibrating. A series of waves of gradually decreasing amplitude will be traced, and evidently if we knew the rate of the fork we could tell how long contact had lasted, within a very minute fraction of a second, by merely counting the waves. This is the principle of the tuning-fork chronograph. The experiment may be varied by blackening the edge of one of the prongs, and drawing a needle along it. The waves are now traced on the fork itself, and if measured, would give the amplitude of vibration at any point.

Experiment LI

Hang the fork by a thread, and strike it gently with a piece of metal. It now gives a shrill tingling sound due to higher rates of vibration which here overpower the fundamental, while in the ordinary mode of excitation they are either not heard at all, or die away very soon. Thick forks give these shrill tones much more easily than thin ones.

Experiment LII

Excite a fork gently, hold it near the ear, and rotate it. In four positions the sound will cease entirely, because

a condensed pulse from one prong and a rarefied one from the other reach the ear together. The curves of silence are hyperboloids, as shown on p. 164.

A similar effect is produced when a fork is revolved over a resonating jar.

Experiment LIII

Clamp a fork so that the plane of vibration is vertical. Sprinkle some fine sand over the upper face, and draw a bow down at about two-thirds of the length from the end. A transverse nodal line is now seen some little distance from this end, and at the same time a note is heard, much higher than the fundamental. The pitch of this and the successive higher notes of a fork are represented by the numbers $6\frac{1}{4}$, $17\frac{1}{2}$, $34\frac{1}{4}$, $56\frac{1}{2}$; the fundamental being reckoned as unity, though they are liable to variation.

When the fork is broad, bowing it in the manner described sometimes causes a longitudinal line to form, bisecting the upper face.

Experiment LIV

Make a narrow cylinder of stiff paper, hold it so as to cover up one of the prongs: the sound is now louder than if both were uncovered. For this experiment the fork must not be on a resonance box. The effect is due to a checking of the interference which results from the opposing movements of the prongs. It is quite possible to notice it by merely shading one of the prongs by a flat sheet of paper when the ear is held in the proper position.

Experiment LV

Make a file-mark in the shank, and while the fork is sounding, press it against a sonometer wire. At a certain

point the wire sings out loudly, but a very small displacement causes the effect to cease. This length of wire would, if sounded alone, be in unison with the fork. A series of measurements may be made, using several forks of known frequency, to prove the law of variation of length.

In this experiment considerable accuracy may be attained by resting the fork on the movable bridge instead of on the bare wire, and moving it five or more times from right to left, and as many in the opposite direction, taking a reading each time.

Example.—

Frequency of fork 320 :

Lengths of Wire giving Maximum Resonance.

Bridge moved from right to left.	Bridge moved from left to right.
529	530
528	533
531	533
528	530
530	531

Mean 530·3 mm.

Frequency of second fork 384 :

Lengths of Wire giving Maximum Resonance.

Bridge moved from right to left.	Bridge moved from left to right.
444	446
442	445
442	443
442	442
441	444

Mean 443·1.

Suppose the pitch of the first fork unknown, we have

$$\frac{n}{384} = \frac{443 \cdot 1}{530 \cdot 3}, \text{ whence } n = 320 \cdot 8.$$

By a simple rearrangement the stretching weight may be made the unknown quantity, and calculated from the formula $W = 2mn^2l^2$, where m is the mass of 1 linear

centimetre, n the frequency, and l the length. The formula itself is derived by an obvious substitution from the one given on p. 27, viz. $n = \frac{1}{2rl}\sqrt{\frac{Tg}{\pi\Delta}}$.

Experiment LVI

Procure two forks accurately in unison. If they are not exactly so, load the higher of them by pressing a small piece of wax on each prong. A musical ear can detect a difference of about two vibrations per second, but in any case the weighting must be varied until when the shanks are pressed together either fork will set the other into sustained vibration.

When the unison is perfect, strike one fork, and hold it very close to the other: the latter will then begin to ring by the aerial impulses alone. If they are both on resonance boxes the effect is almost instantaneous, and they need not be close together. It must be remembered that several hundred impulses are given every second, and each one at the exact moment when it produces most effect. A *very* slight departure from unison causes the passive fork to vary in intensity in the same manner as the wire in Expt. XXI.

Experiment LVII

Instead of holding the active fork stationary, move it quickly to and fro in the immediate vicinity of the other: the latter is not now excited, because the motion of the hand causes the waves to strike at one time too rapidly, and at another too slowly. Dr. Koenig has pointed out that this motion may supply what is wanting in frequency, *i.e.* if one fork describes 256 vibrations and another 258, an approach of the former at a rate which will add two more waves per second will bring about the sympathetic

effect. The rate can be readily calculated, for the wave-length is, as usual, = velocity ÷ frequency = about 130 cm. in the case supposed.

Experiment LVIII

Press the handle of a vibrating fork on a Chladni's plate; it resounds far less satisfactorily than a wooden board, because the plate is probably not capable of emitting the right note under any circumstances, and so does not take up the motion, while wood is more compliant. The unyielding nature of the metal also causes a jarring which spoils the resonance.

The distortion of a tuning-fork under different pressures may be investigated by the method described in vol. i. of this series, p. 185. It will be found to obey the usual law that the deflection is proportional to the bending force. This is the reason why the vibrations are regular, as will be explained under Harmonic Motion.

Experiment LIX

Hold the prong of a vibrating fork in a Bunsen flame, the sound becomes louder, as if the tube or flame acted as a resonator. Since the effect takes place almost equally well over a Fletcher's ring burner, this can hardly be the whole explanation.[1]

Experiment LX

To calculate the Elasticity of Steel from the Dimensions of a Fork and its Frequency

The formula is, in the first instance, $n = \frac{\cdot 164 v\theta}{l^2}$ where v is the velocity of sound in steel, l the length of the prongs,

[1] See *Nature*, 5th December 1878.

and θ their thickness in the plane of vibration. We shall see hereafter that $v = \sqrt{\frac{E}{\Delta}}$, where E is the quantity sought, and Δ the density, and by rearrangement we have $E = \frac{n^2 \times l^4 \times \Delta}{(\cdot 164)^2 \times \theta^2}$.

Example.—A fork for which $l = 14$ cm. $\theta = \cdot 6$ cm. and $\Delta = 7 \cdot 8$, made 256 vibrations per second. By substitution—

$$E = \frac{256^2 \times 14^4 \times 7 \cdot 8}{\cdot 164^2 \times \cdot 6^2} = 20 \times 10^{11} \text{ nearly.}$$

The usual value is from $20 \cdot 2$ to $24 \cdot 5 \times 10^{11}$.

Experiment LXI

To find the Energy of Vibration of a Tuning-fork when its Amplitude is given

The quantity desired is $\frac{1}{8} m \frac{\pi^2}{t^2} (2\eta)^2$, where m is the mass of a single prong, t the period, and η the amplitude. For the two prongs together it is twice this amount.

Example.—The fork used in the last experiment had a breadth of $1 \cdot 1$ cm., and its amplitude at a certain moment was $\cdot 05$ cm. The total energy is then $\frac{1}{4} \times 14 \times \cdot 6 \times 1 \cdot 1 \times 7 \cdot 8 \times 256^2 \times \cdot 1^2 \times \pi^2 = 4 \cdot 06 \times 10^3$ ergs.

The amplitude diminishes in a logarithmic ratio represented by $e^{-\frac{1}{2}\kappa t}$, while the energy diminishes at the rate κE per second, κ being a constant. It is therefore possible to calculate the extent of swing of a fork just before it becomes inaudible by finding how many seconds it can be heard after a certain moment when its amplitude is known. How excessively minute this is may be gathered from Expt. XLVIII. p. 66. It has been observed that the minimum amount of energy required to excite the eye and ear is of the same order of magnitude; *i.e.* that the amount supplied per second by the faintest light that can

be seen and the faintest sound that can be heard, are not very different.[1]

Experiment LXII

Instantaneous Views of a Vibrating Fork

One method which may be used is the same as that described on p. 32, but little can be gathered from it.

It is more instructive to make a stroboscope from a disc of cardboard about 15 cm. in diameter with some 18 or 20 radial slits cut out of it, leaving a continuous rim so as not to spoil its rigidity. A metal disc is preferable. This is mounted on a horizontal axis on a whirling table, and the fork viewed through the slits as they cross the line of sight.

If they succeed one another at the same rate at which the fork vibrates the prongs appear stationary, because they are always seen at the same period of their swing; if a little slower or faster, they open and close alternately. Looking at a horizontal fork, when the rate is properly adjusted, the appearance shown in Fig. 30 is observed.[2]

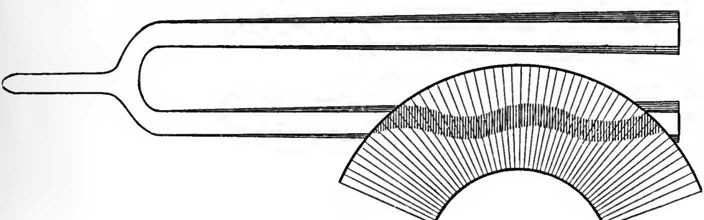

Fig. 30.

When the wheel gains on the fork, the waves move outwards in the direction of its motion, and *vice versa*. The explanation is that we see short sections of the prong along the portion covered by the wheel, and each section moves

[1] Rayleigh, *Theory of Sound*, vol. ii. p. 13, and elsewhere.
[2] Whiting, *Physical Measurement*, Boston, 1891.

with a harmonic motion, hence the whole appears as a wave. This will be more clear after Chapter IX. has been read.

Experiment LXIII

Melde's Experiments

Required.—Large tuning-fork; silk thread; bow; clamp; wax; small wire hook. Arrange as in Fig. 31, but with the plane of vibration at first horizontal.

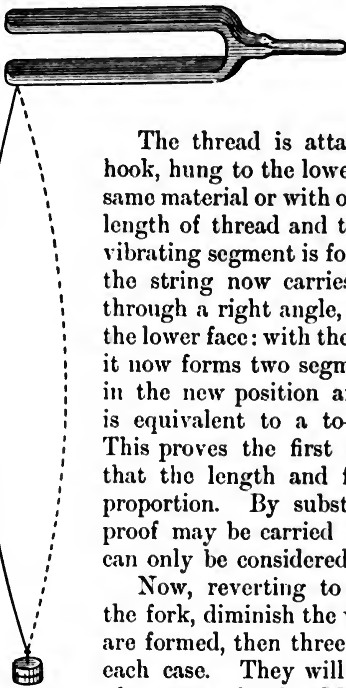

Fig. 31.

The thread is attached by soft wax, and a hook, hung to the lower end, is loaded with the same material or with ordinary wax. Adjust the length of thread and the load so that only one vibrating segment is formed. Find what weight the string now carries. Next turn the fork through a right angle, and fasten the thread to the lower face: with the same length and tension it now forms two segments instead of one, for in the new position an up-*or*-down oscillation is equivalent to a to-*and*-fro one in the old. This proves the first of Mersenne's laws, viz. that the length and frequency are in inverse proportion. By substituting other forks the proof may be carried further, but the method can only be considered a fancy one.

Now, reverting to the former position of the fork, diminish the weight till two segments are formed, then three, and find the tension in each case. They will be nearly in the ratio of $1 : \frac{1}{4} : \frac{1}{9}$, for by Mersenne's third law the number of vibrations is directly as the square root of the tension. The actual rate of vibration does not vary, being simply that of the fork; hence the length of

the segments is directly, and the number of segments inversely, as the square root of the tension. The above experiments are known as Melde's: they were suggested by the behaviour of a string tied across the mouth of a bell jar.[1] In a more elaborate form of apparatus, a wire, hanging by the side of a resonance box (like a vertical sonometer), is attached to the prong of a large fork maintained electro-magnetically, and the two sounds (of the fork and wire) can be heard together.

Experiment LXIV

Determination of g by a Tuning-fork

Apparatus required.—See Fig. 32. B is a wooden base; A is a standard with a cross-bar C near the top; n and n_1 are pins from which a smoked glass plate G hangs by a cotton thread; F is a tuning-fork of known frequency, say 256, the handle of which fits firmly in a hole in the base; p is a style attached to one of the prongs, and a counterweight (not shown) is fastened to the other to preserve the balance. The positions are so arranged that the style is just in contact with the glass: the fork is then bowed and the plate allowed to fall by burning the thread. Count the number of waves traced on the glass and measure the length of the whole set; thus we find not only the space travelled, but

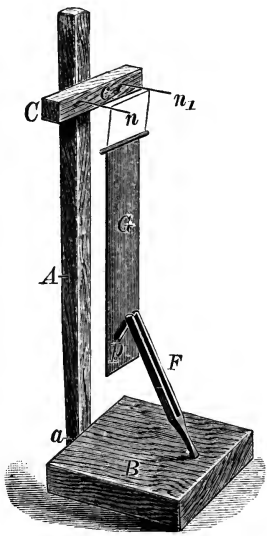

Fig. 32.

[1] See Tyndall, *Sound*, chap. iii. and *Proc. Roy. Inst.* June 1866.

also the time taken. Suppose, for instance, that 33 waves are found in a space of 8 centimetres, then from the formula $s=\frac{1}{2}at^2$ we have $a=\frac{2s}{t^2}=\frac{16}{\left(\frac{33}{256}\right)^2}=\frac{16\times256^2}{33^2}=963$ cm. per second, instead of 981.

The waves are difficult to count at the commencement, being very much crowded together. A better way is to measure them in groups of 20 or 30 when they have begun to spread out, we then find that there is a constant increase in length: for example, if the first 20 take up a certain space, the second 20 take up a space greater by a certain amount; for the third 20 the increment is the same as before, and so on. If l_1 be the total length of the first n waves and l_2 the length of the next n waves, and t the time interval, then $g=\frac{l_2-l_1}{t^2}$. This application of a tuning-fork is due to Professor Boys.

A more elaborate apparatus, in which a tuning-fork is used as a chronograph to measure the time taken by an iron ball to fall through a height of 3 m. or more, is described in vol. i. of this series, Lesson LIII.

Determination of the Rate of a Fork

This is accomplished by means of a pendulum beating seconds (or some known interval), which carries a tapering piece of platinum underneath, and at the lowest point of its swing makes contact with a small quantity of mercury in an iron cup. The pendulum is a conductor, and is connected with a battery and with the primary wire of an induction coil. At every contact the current passes for an instant, and is immediately broken, then an induced current flows through the secondary wire. This latter is in circuit with the fork and a metal drum covered with blackened paper. The fork carries a brass wire style, and traces a sinuous line on the drum, which is kept slowly

revolving. The induced current causes a spark to pass from the style, and makes a fine dot at some point of the curve. On counting the number of waves and fractions of a wave between two dots the frequency of the fork is obtained at once. It is better to take the mean of two or an even number of pendulum beats, because if the mercury is a little on one side the intervals will be too small and too great alternately. The method is due to Professor Mayer.[1] It may be used to compare the frequencies of two forks, by making them both draw traces on a revolving cylinder covered with blackened paper, and counting the number of waves comprised within a certain limit of space when the paper is unrolled. Much use is made of the tuning-fork as a chronograph in connection with the flight of projectiles and other investigations when a very short interval of time has to be measured.

[1] See also A. A. Mitchelson, *Phil. Mag.* Feb. 1883.

CHAPTER VIII

ORGAN PIPES

FOR musical purposes organ pipes are made either of wood or metal, of various lengths up to 32 (reputed) feet, and with diameters to correspond. Wooden ones are square or rectangular (sometimes even triangular) in section; those of metal are circular, but not always either cylindrical or straight; some are open at the far end; some closed by a leather-faced plug. Those in which the air is thrown into agitation by being blown against a lip are called flue or flute pipes, to distinguish them from reed pipes, where a small flexible tongue of metal is the prime mover. These latter are always open, with one partial exception, where the end is closely shaded by a metal lid. The appearance of an ordinary wooden pipe is shown in Fig. 33. The air enters by a tube called the "foot," thence it is guided so as to flow in a thin sheet across the "mouth," or opening in the front face of the pipe; it next strikes against the "lip," and begins to flutter like a reed, performing its outward and inward oscillations in time with the aerial pulses in the tube.[1] The *pitch* of the note is dependent chiefly on the length of the tube, whether it is stopped or open, and upon the force of the air and its temperature. Its *quality* is influenced in a greater or less degree by every conceivable circumstance, *e.g.* the material, shape, and

[1] See Helmholtz's *Sensations of Tone*, Appendix XIX.

"scale" of the pipe (*i.e.* the ratio of the cross-section to the length), the form and position of the "languid" or internal edge of the slit through which the air passes, etc. etc.

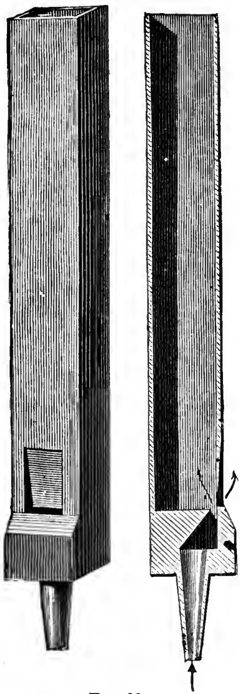

Fig. 33.

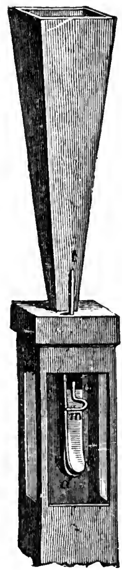

Fig. 34.

The length of a pipe is measured from the lip to the far end if open, or to the plug if stopped.

In reed pipes (Fig. 34) the air has to pass between the reed *m* and the slot *ab*, which it covers. Looking at the direction of the air, it would appear calculated to drive the

reed firmly against the edges, and keep it there, but this is not what happens. The aperture is not quite closed, because the reed is slightly curved, and a little air gets past, thus a vibration is started and maintained so long as there is a sufficient current. The reed shown in Fig. 34 is a "striking" one, *i.e.* it is broader than the slot, and comes in contact with it at every vibration. No other kind is used in organs, but in harmoniums and reed instruments which have no pipes, free reeds are employed, the slot being wide enough to enclose the reed and leave a narrow aperture all round. The piece of bent wire shown in the figure can be worked up and down so as to tune the reed by altering the length of the vibrating portion.

The length of pipe is not without influence on the note of a reed: if it is at an antinode the oscillations are unaffected, but if not, and especially when at a node, its pitch is lowered.

As regards the tuning of organ pipes, open wooden ones are usually provided with a flat plate of organ-pipe metal let into a saw-cut at the end, and by bending this so as to shade the orifice more or less, a slight amount of flattening is produced; or if the pipe is too flat originally a small piece is sawn off so as to reduce its length.

Open metal pipes are tuned by moulding the end with a wooden cone, widening or contracting it as may be necessary: in stopped pipes a slight adjustment of the plug produces the required result.

When an open flue pipe is blown, some air escapes from the far end, but the bulk of it flows past the lip.

There is a close resemblance between the air in a flue pipe, whether stopped or open, and a rod or wire vibrating longitudinally; in both the material is alternately condensed and dilated, and the same arrangement of stationary waves, with their nodes and loops, exists in the one case as in the other.

The far end of a closed pipe is always a node, like the

fixed end of a rod, because changes of density can take place there, but the excursions of the particles are prevented. At the lip there is always an antinode, because changes of density are balanced from without, but there is no check on the amplitude. In an open pipe the far end, or more accurately, a point a little further out, is always an antinode for the same reason. A node and an antinode must always follow each other successively, so that in an open pipe there must be at least one node; it is found to be rather more than half-way along, and the smallest fraction of a wave that it can contain is one half. In a closed pipe, on the other hand, it is a quarter of a wave. In other words, when a pipe is blown with a moderate degree of force, the wave-length of the sound it gives out is twice the length of the pipe if it is open, and four times if closed. Hence the note in the latter case is (nearly) the lower octave of what it is in the former when the pipes are the same length. This may be represented graphically as in Fig. 35, the corresponding portions of a transverse wave being given alongside. It will be evident that if a closed pipe vibrates in any other way, it must have two nodes as in II, or three as in III, and so on; while an open one must split up as in II′, III′, etc. The condition of the air as to direction and velocity is explained on p. 96. In II there are three quarter-waves, and in III five quarter-waves; and as the rates of vibration are inversely as the lengths of the waves, these modes will involve respectively three and five times the rapidity of vibration of the fundamental. They are produced simultaneously with the latter as a general rule, but by increasing the blast of air the lower notes disappear in turn, and the higher ones predominate successively.

Similarly, in open pipes the frequencies induced by different pressures of air will be as 1 : 2 : 3, etc. These relations were first established by Daniel Bernouilli. A doubly-stopped pipe, when tapped, has the same pitch as a

doubly-open one, because, having a node at each end, the least fraction of a wave which it can contain is one-half.

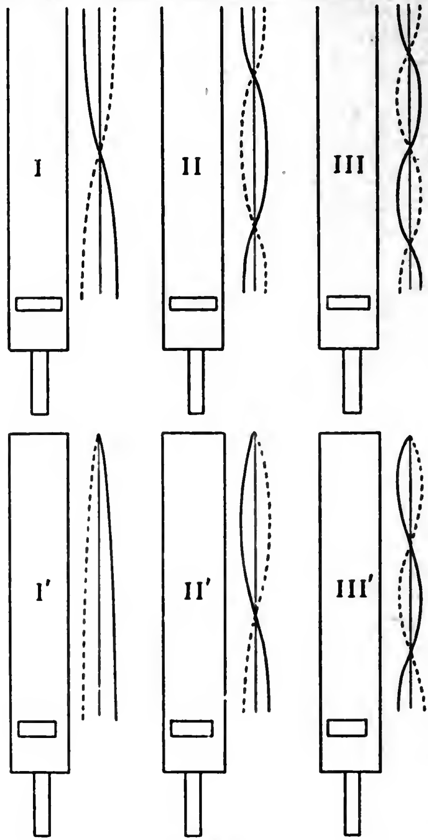

FIG. 35.

It must be observed that the pitch of an organ pipe is lower than would be calculated from its length and the

density and elasticity of the air, but no exact correction can be applied, because much depends on the relation of its various parts, and upon minute details which fall within the province of an organ-builder, for whom practice is independent, and in some respects in advance, of theory.

Experiments with Organ Pipes

Required.—Various organ pipes, open and closed ; tuning-forks ; half-metre scale.

Experiment LXV

Take an open pipe and blow very gently into it: a rustling of air is heard, along with a faint note much above the proper one. The rustling is such as would be produced by a current of air blowing against a lip independently of the pipe.

Experiment LXVI

Blow steadily but not too hard: it now gives its proper or fundamental note. A moderate increase in the force of air raises the pitch slightly. Hold the palm of the hand a short distance beyond the open end of a 2-foot pipe; the agitation of the air will be distinctly felt. By partially closing the aperture the note will be lowered in pitch, and the same result follows when a finger is held near the lip of the pipe. Close the far end altogether with a flat plate, no satisfactory note can be obtained; but, leaving a small aperture open, it gives a note slightly above the lower octave of the fundamental. A squeezed handkerchief makes an effective plug.

Experiment LXVII

Increase the force of the current considerably; a note an octave higher than the last will be heard, due to the

pipe taking its second mode of division, and with a still greater force the third mode of division will be induced, and the note has three times the rate of vibration of the fundamental, its position in the scale being the Fifth above the octave. The intervals, however, are not quite exact.

Experiment LXVIII

Hold a vibrating tuning-fork near the lip of a pipe giving the same note: a powerful resonance is produced.

Experiment LXIX

Blow the pipe with coal gas or hydrogen: it gives a shriller note than before (in the latter case nearly two octaves higher). With carbonic acid, on the other hand, the note is lowered.

Experiment LXX

Using a closed pipe instead of an open one, observe that the note is quieter and sweeter than before. Alter the position of the plug: the note rises and falls according as it is pushed in or out. Remove the plug altogether: the pipe now may fail to "speak" at all. Replace the plug and blow with gradually increasing degrees of force, the first higher note to be obtained is due to three times the rate of vibration of the fundamental, and the second to five times.

Experiment LXXI

Nodes and Antinodes in an Organ Pipe

Required.—Small organ pipe, with a plug which can be moved to any point inside; sonometer.

Sound the open pipe so as to give its fundamental note,

and tune the sonometer wire to it. Now, insert the plug and move it along till the note, which at first sinks nearly an octave, is once more in tune with the wire. Find the position of the face of the plug from the end; in theory it should be half-way down, but in practice it is a little further out. The pipe had originally an antinode at each end, and a node in the middle; but when there is a plug at a node the rate of vibration is not altered. Nor is the fundamental altered when a hole is made at an antinode.

Experiment LXXII

Make the pipe give its first higher note by overblowing. With an open pipe this is the upper octave, and, as we have seen, it now contains two nodes; hence, as the plug advances, there are two positions where the original note will be restored. Find those positions, using the sonometer wire as a standard, and compare them with the theoretical ones.

Experiment LXXIII

Required.—Open organ pipe with one side made of glass; small ring with thin membrane stretched over it, and suspended by a string; fine sand.

The pipe being in a vertical position, sound it as usual, and having put a little sand on the membrane, lower it gradually. The sand is at first agitated considerably, but quietens down as it approaches the middle. There must be a space left between the ring and the interior of the pipe, otherwise it is apt to create a node wherever it is.

No satisfactory determination of the positions of the nodes and antinodes in an overblown pipe can be made in this way, for various reasons. Dr. Koenig, however, has pursued the inquiry with a large organ pipe 2·3 m. long by 12 cm. square, laid horizontally in a trough of water,

and having a slit all along the lower side, through which one branch of a U-tube was passed. The other branch was connected with a small water gauge, and when the pipe sounded its 8th note, the nodes were found at 173, 488, 808, 1122, 1438, 1747, and 2018 mm. from the lip. Thus the spaces which, according to theory, should be all equal, were respectively 315, 320, 314, 316, 309, and 271 mm.

EXPERIMENT LXXIV

Measurements of Organ Pipes

Required.—A number of stopped and open pipes; metre scale.

Obtain the lengths and diameters of all the pipes, measuring the stopped ones from the lip to the face of the plug. Compare the frequencies obtained from the musical interval between them (see p. 147) or otherwise, with the inverse ratio of the lengths. Also compare the frequencies with those calculated from the formula $n = \frac{8250 + 15t}{L + \cdot 8r}$ for a stopped pipe, and $n = \frac{16500 + 30t}{L + 1 \cdot 1r}$ for an open one, where L is the length, r the radius and t the temperature centigrade. For a rectangular pipe the radius of the equivalent circle may be taken: if the sides are a and b this is $\sqrt{\frac{ab}{\pi}}$ or $\cdot 56\ \sqrt{ab}$. It will be found that the theory is somewhat widely departed from.

Example.—Three open wooden pipes whose dimensions were respectively 60·4 × 4·3 × 4·3 cm., 39·2 × 3·3 × 3·3, and 28·6 × 2·7 × 2·7 gave the notes c', g', c'', which are in the numerical ratio of 2 : 3 : 4. The radii of the equivalent circles being 2·4, 1·8, and 1·7 cm. respectively, we get the effective lengths 60·4 + 2·6 = 63·0, 39·2 + 2·0 = 41·2, and 28·6 + 1·9 = 30·5 : hence the second pipe is ·8 cm., and the third 1 cm. shorter than if they had the theoretical lengths.

The formulæ given above are derived from the fractions of a wave which a stopped and an open pipe respectively contain. In the former case (taking the simplest mode of vibration), it is one quarter of the wave-length. But wave-length × frequency = velocity of propagation = 33000 cm. per second, with a correction of 60 cm. per second to be added for each degree above zero. Hence $4n(L + \cdot 8r) = 33000 + 60t$, from which $n = \frac{8250 + 15t}{L + \cdot 8r}$. A comparison of this result with the actual frequency will show that little reliance can be placed on it.

Experiment LXXV

Compare the lengths of stopped and open pipes which give the same note. In theory they should be as 1 : 2, but in practice the ratio is less; that is to say, an open pipe may be as much as 2·6 times as long as an equivalent stopped one, the amount varying with the size and scale of the pipes.

CHAPTER IX

SIMPLE AND COMPOUND HARMONIC MOTIONS

LET APB (Fig. 36) be a circle whose centre is O, and let a point P move round it with uniform velocity. Then X, the foot of the perpendicular, or more concisely, the projection of P on AB, moves between A and B with a simple harmonic motion. The motion of P, being always directed along the tangent, will be equal to that of X when the latter passes through O, but at all other points it will be greater. It is known from Dynamics that a constant acceleration towards the centre is necessary to keep P moving as described, and if this be represented by OP, then the resolved parts of it are OX and PX; in other words, the acceleration of X is always proportional to OX. Hence when a particle describes a simple harmonic motion, the force acting on it is directed towards the middle of its path, and is proportional to its distance from that point. The definition is sometimes given in this form instead of the other.

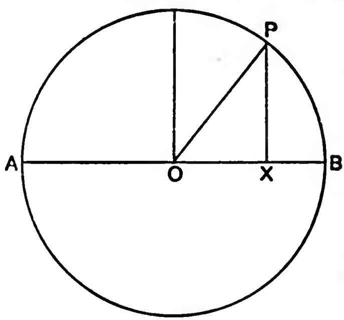

FIG. 36.

Since OP, PX, and XO are respectively perpendicular to the motions of P, of X, and of the other component, the triangle OPX is the triangle of velocities, so that the velocity of X at any moment is represented by PX. The words period and amplitude have the meanings already described: the angle POB is called the *phase* of the vibration.

It is worth while pointing out that the motion of a piston driving a fly-wheel is nearly simple harmonic, because the latter moves with a nearly uniform velocity, and the connecting rod, though it does not remain always parallel with itself like the perpendicular PX, still the angle it moves through is not great.

A pendulum presents the most familiar example of simple harmonic motion. That this is so is easily proved as follows. Let AB (Fig. 37) be the position of rest, and AC one making a small angle with it, so that the arc and chord BC may be supposed to coincide. Draw BD horizontally, and CD vertically. Then at C the particle begins in effect to run down a plane whose inclination CBD = BAC, and its acceleration is $g \sin \mathrm{CBD} = g \sin \mathrm{BAC} = g \times \frac{\mathrm{BC}}{\mathrm{BA}}$. Here the only variable is BC, so that the force is proportional to the displacement, when the arc BC is sensibly straight.

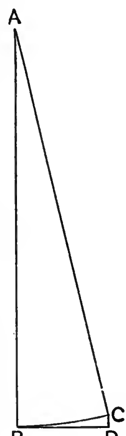

FIG. 37

We have pointed out that this principle applies to stretched wires, tuning-forks, reeds, and sound-producers in general; it is no less true that the oscillations of the particles of air or other medium when conveying a simple sound-wave, *i.e.* one produced by a pendular vibration, follow the harmonic law.

When a heavy particle is suspended by a flexible string, and caused to move with uniform velocity in a hori-

zontal circle, thus making a conical pendulum, its motion, as seen from a point in the plane of this circle, is nearly simple harmonic, and would be quite so but that the line joining the eye to the particle moves through a small angle. The time of describing the circle is the period (double vibration) of a pendulum of the same length, and the uniform velocity is that which the bob would have at the lowest point of its swing.

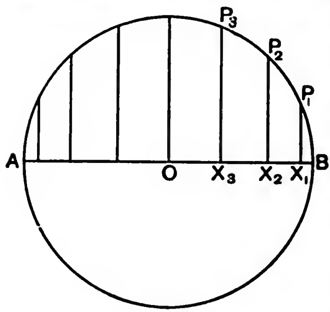

FIG. 38.

From Fig. 38, where the circle is divided into a number of equal parts, we see that, since BP_1, P_1P_2, etc., are described in equal times, therefore BX_1, X_1X_2, etc., are also described in equal times. This observation is of importance in connection with the curves shown in Fig. 51.

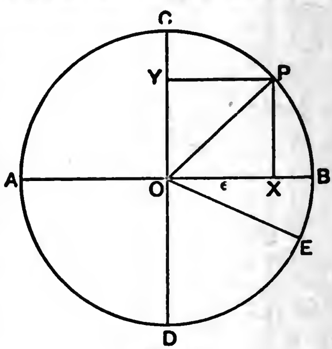

FIG. 39.

It is easy to calculate the position of the particle at any moment of its path by reckoning where it would be on the circumference if it completed a revolution in the same period, and drawing a perpendicular to the line of motion. If the time be reckoned, not from the moment when the particle is at B, but from some other moment when it is at E, say, then the interval between this and the arrival at B is called the *epoch*. It is represented in Fig. 39 by the angle ϵ, though it is really a time interval.

The period of a harmonic vibration is independent of the amplitude. For in Fig. 39 let the acceleration at the point X, whose distance from O may be called x, be denoted by μx. Then since the force at this point is proportional only to x, it follows that μ must be a constant. Let $OP = r$ and let T be the time of executing a complete to-and-fro vibration. Then if v be the constant velocity of P in the circle, the acceleration is well known to be $\frac{v^2}{r}$. But the total path of P is $2\pi r$, and is described in time T, so that $v = \frac{2\pi r}{T}$. Substituting, we get the acceleration $= \left(\frac{2\pi}{T}\right)^2 r$. The horizontal component of this at the point X is $\left(\frac{2\pi}{T}\right)^2 x$, and equating this to μx gives $\mu = \left(\frac{2\pi}{T}\right)^2$ or $T = \frac{2\pi}{\sqrt{\mu}}$, an expression in which the radius or amplitude does not occur. This principle is of very great importance in music, for otherwise there could be no variation in the intensity of sounds without an alteration of pitch.

Since in the above figure everything is symmetrical with respect to both diameters, it is clear that the point Y also describes a harmonic motion along CD; or in other words, that two harmonic motions at right angles, and with a phase-difference of 90° (*i.e.* one being at a maximum while the other is a minimum), compound into uniform circular motion. This may also be proved mechanically as on p. 94.

If θ denote the angle POB, which, as before, is the *phase* of the vibration, then PX or OY (usually denoted by y) $= r \sin\theta$ and $OX = r\cos\theta$; but if θ be measured not from OB, but from some line OE which makes an angle ϵ with it, then $x = r\cos(\theta - \epsilon)$ and $y = r\sin(\theta - \epsilon)$. The angle θ is here proportional to the time, reckoned from an arbitrary starting-point. If t denote the time of describing the

angle θ, then since T is the time of describing 2π, we have

$$\frac{\theta}{2\pi}=\frac{t}{T} \text{ or } \theta=\frac{2\pi t}{T}.$$

Substituting in the formula, we have $x=r\cos\left(\frac{2\pi t}{T}-\epsilon\right)$, by a slight alteration this becomes $x=a\cos(\omega t-\epsilon)$, where a stands for the amplitude and ω for $\frac{2\pi}{T}$. This expresses that x is a simple harmonic function of t. In its most general form it becomes $y=a\cos x$, and represents the curve shown in Fig. 41, which is compounded of a simple harmonic motion in a vertical direction with a uniform motion in a horizontal one. Whenever $\cos x$ vanishes, $y=0$, and when it $=1$, $y=a$, so that the curve cuts the axis of x at regular intervals, and is above and below it alternately.

Experiment LXXVI

Combination of Harmonic Motion with uniform motion at right angles

Required.—For this and similar experiments with a pendulum the apparatus shown in Fig. 40 is very useful.[1] It consists of a wooden frame, a little over 1 m. in height, the cross-bar of which is bored with two holes, through which a cord passes, pulled into a V-shape by a leaden disc. This disc is about 7 cm. across and $1\frac{1}{2}$ cm. thick, and has a conical hole in the middle, large enough to support a tin or glass funnel, with a small aperture at the end. The use of the lead is to keep the funnel steady and prevent wobbling. The cord passes through a wooden peg at the top, and is coiled once or twice round it, so that the funnel may be adjusted till it nearly touches the base when hanging vertically. Sand is poured into the funnel, and issues in a thin stream from

[1] See Mayer's *Sound*, "Nature Series."

the tapering end. When the pendulum is pulled aside at right angles to the plane of the frame, it executes a harmonic vibration in this direction. A piece of flat card is now drawn with a uniform velocity between the uprights,

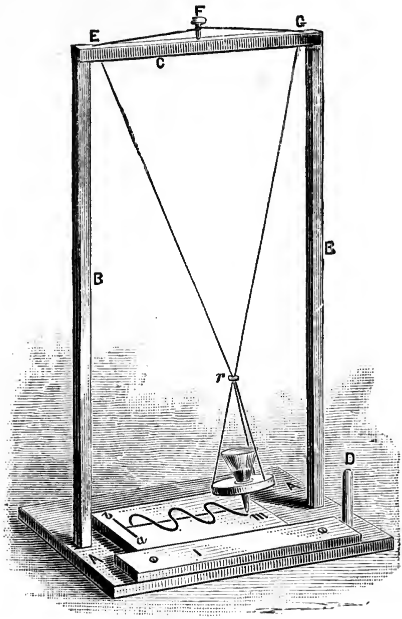

FIG. 40.

and the sand falls upon it in a harmonic curve precisely similar to that drawn in Fig. 41.

The principle here employed is that of self-registering instruments in general, where a pen, moving in any manner whatever along a certain straight line, traces a curve on a sheet of paper drawn with uniform velocity at right angles to that line.

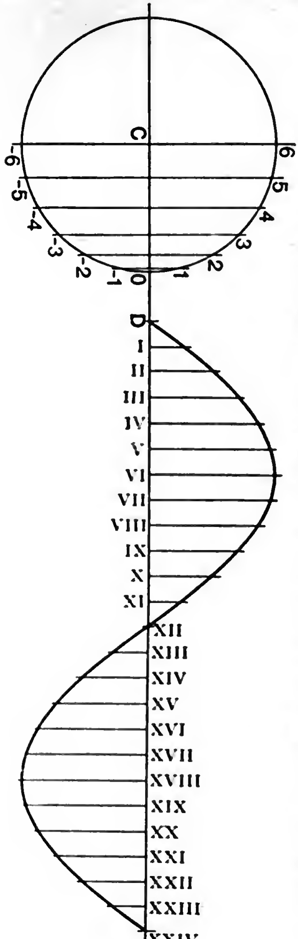

Fig. 41.

Experiment LXXVII

Try to make the sand lie in a circle by moving the paper with the hand. It will be found that the only way to do this is to exactly reproduce the motion of the pendulum, but in a direction at right angles to its swing (see also p. 91).

Graphical Method. — The curve described in Expt. LXXVI. may be drawn with greater accuracy by the following process :—

Draw a semicircle with centre C, divide it into, say, 12 equal parts, and draw perpendiculars on OC (Fig. 41). Produce this latter indefinitely; take any point D and set off 24 equal spaces, numbering them as shown. From I draw a perpendicular equal to the height of 1 above OC, and so on, up to VI. At VII take the same height as at V, and proceed in regular order to the end. Draw a curve freehand through the points thus obtained. It is called a "sinusoid," because it shows the variations in the sine of an angle as it increases from 0° to 360°. It con-

sists of a series of ridges and furrows, or crests and troughs, and is the elementary form of a wave. Any degree of steepness or flatness can be given to it by altering the distance from D to XXIV; when this is equal to the circumference of the auxiliary circle it has its simplest form.

The curve is a kind of analysis of harmonic motion, and by the method of construction all the spaces between the successive points are described in equal times. Comparing it with the one in Fig. 40, it shows very clearly how, when the pendulum is stationary and the paper alone moving, the tangent at that point, the highest or lowest in Fig. 41, is parallel to the axis; and that at other points it moves along the diagonal of a rectangle, of which one side is constant, representing the uniform velocity of the paper, and the other takes all values between zero and the maximum velocity of the pendulum. The value of the tangent where the curve crosses the axis is the quotient of the latter of these velocities by that of the paper. Where the pendulum stops, as it does at the highest and lowest points, the tangent becomes zero, but it cannot become infinite, because it can never cut the axis at right angles.

We see also that the rate of change in the value of the tangent, or the alteration of curvature, is greatest just before and after the extreme limits of the curve; in other words, the change in the acceleration is a maximum at these points and vanishes in the middle.

The wave-length is the distance in a straight line from any point on the curve to the next point which is in the same phase, *i.e.* in a similar position but on the succeeding branch. When a row of equidistant particles is displaced so that each has a simple harmonic motion of the same amplitude and period, but differing by a constant interval in the time of arriving at the same phase, they all lie at any moment on the surface of a wave such as we have been considering. If we draw on the same axis a second curve

similar in all respects to the first, but displaced a quarter of a wave-length, it represents the *velocities* of the particles; this is zero when they are at the limit of a swing, and a maximum in passing through the middle point.

If we suppose the particles to be displaced so that each

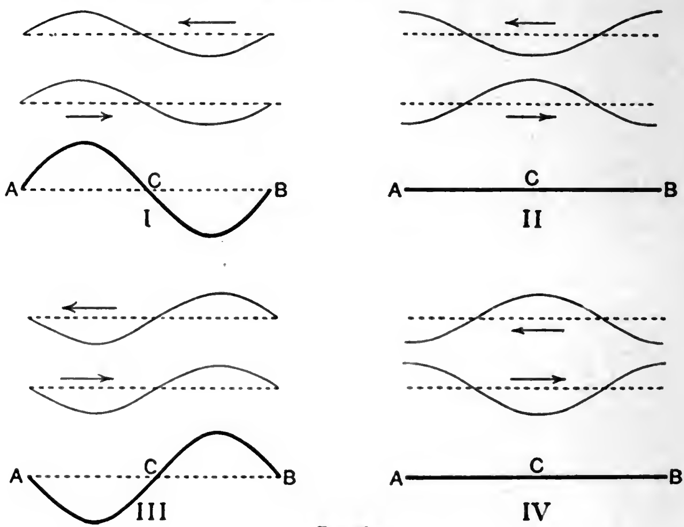

FIG. 42.

moves at right angles to its former path, but with no other change, then they will form a series of condensations and rarefactions such as belongs to a simple sound-wave in an elastic medium. The displaced curve drawn as above now represents not only the velocities of the particles, but the degree of condensation and rarefaction.

When a system of such waves, longitudinal or transverse, is met by a similar system travelling in the opposite direction, a series of stationary waves is formed as in

Expt. XI. For let them meet as in Fig. 42, I, then their combined effect is shown at ACB, with an amplitude double of either of the single ones. In a quarter of a period the position of things is shown in II; the surface is perfectly level for the moment (in the case of longitudinal waves the medium is at its ordinary density). Nos. III and IV follow in order. The middle and the two terminal points then remain permanently at rest, and are called *nodes.* The other terms, antinode, wave-length, etc., have been already explained.

In longitudinal waves the changes of density are greatest at the nodes, and are nothing at the antinodes.

The analytical investigation of progressive and stationary waves of this type may be found in Everett's *Vibratory Motion and Sound,* the *Encyclopædia Britannica,* Art. "Mechanics," Deschanel's *Natural Philosophy,* etc.

Experiment LXXVIII

Reproduction of Harmonic Motion from the Harmonic Curve

Apparatus required (see Fig. 43).—Sheet of paper AB with a harmonic curve traced upon it: piece of cardboard

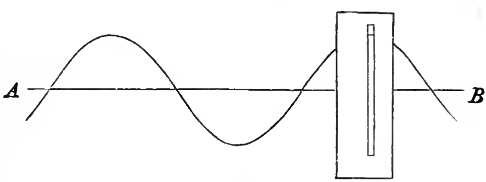

Fig. 43.

in which a slit is cut, which must be at least as long as the curve is broad.

Hold the cardboard so that the slit is over some part of the wave, and at right angles to its axis. Keep-

ing it stationary in this position, draw the paper behind it at a uniform rate and the section of the curve seen through the slit will move up and down harmonically. The process is of course an inversion of that which gives the wave from a simple harmonic motion. If AB be wrapped round a cylinder, of such a diameter that the two ends of the curve meet each other, then the up-and-down harmonic motion could be made continuous.

It will be found very useful to cut a piece of cardboard along the curve, so as to make a "template" from which a trace can be taken at any time.

Superposition of Two Harmonic Curves in the same direction

Graphical Method.—Draw upon any line AB as axis the two curves which it is desired to compound. These are shown in figure 44 by the faint lines. Draw at right

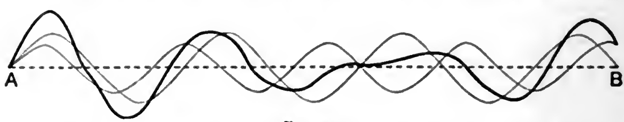

Fig. 44.

angles to AB a considerable number of lines, preferably at equal distances, and when both curves are above or both below AB, add together the portions of the perpendiculars cut off on the successive lines: if one curve is above and the other below, subtract them. Set off these distances, one after another, to the limit prescribed. Trace the compound curve freehand through the points thus obtained: notice the difference between it and the primary curves. If the wave-lengths are commensurable, the curve will repeat itself after a certain interval. The process may of course be performed on any number of curves having the axis AB (see also p. 159). It is important to notice the effect of a difference of phase. In Fig. 45, for instance, the two

upper curves are combined first for a phase difference of 0°, and secondly for one of 90°. The resulting curves would not be suspected to have the same primaries. This consideration assumes very great importance in connection with auditory sensations. When two aerial wave-systems

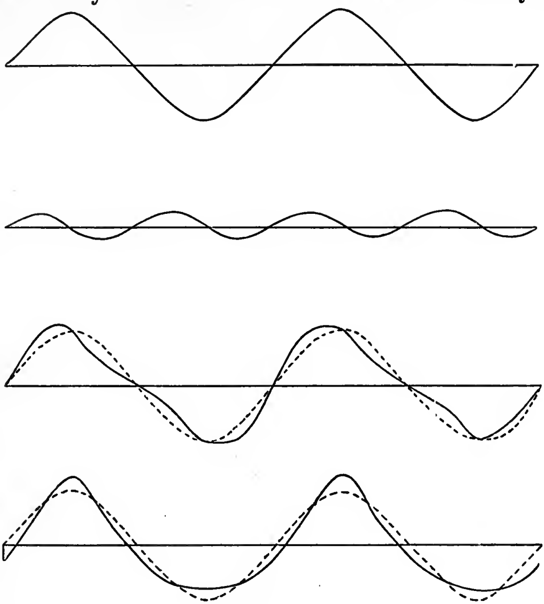

Fig. 45.

are superposed, does the ear perceive any difference such as Fig. 45 would indicate, according to their modes of meeting, or is the sensation the same for all? The latter appears to be in accordance with experience, though the contrary is affirmed by Dr. Koenig (see Appendix III.).

Exercise.—Draw a curve compounded of two harmonic curves, one having an amplitude half that of the other, and differing from it by $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ in phase.

Combination of Two Harmonic Motions at Right Angles—Wheatstone's Kaleidophone

"The apparatus for exhibiting these experiments consists of a circular board about 9 inches in diameter, into which are perpendicularly fixed, at equal distances from the circumference and from each other, three steel rods, each about a foot in length (Fig. 46). The first rod is cylindrical, $\frac{1}{10}$th of an inch in diameter, and is surmounted by a spherical bead which concentrates and reflects the light which falls upon it. The second is a similar rod, upon the upper extremity of which is placed a plate moving on a joint so that its plane may be rendered either horizontal, oblique, or perpendicular: this plate is adapted to the reception of the objects, which consist of beads differently coloured, and arranged on pieces of black card in symmetrical forms. The third is a four-sided prismatic rod, and a similar plate is attached to its extremity for the reception of the same objects. Another rod is fixed at the centre of the board; this is bent to a right angle, and is furnished with a bead similarly to the first-mentioned rod. A small nut and screw is fixed to the board near the lower end of the first rod, in order by pressing upon it to render occasionally its rigidity unequal. A hammer softened by a leather covering is employed to strike the rods, and a violin bow is necessary to produce some varieties of effect.

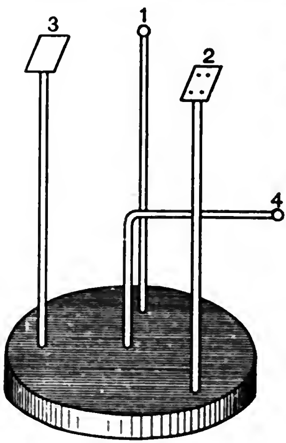

FIG. 46.

No. 1. On causing the entire rod to vibrate, so that its lowest sound is produced, as it is seldom that the motions of a cylindrical rod can be confined to a plane, the vibrations will almost always be combined with a circular motion. When the pressure on the fixed end is exerted on two opposite points, and the rod put in motion in the direction of pressure, the following progression in the changes of form will be distinctly observed. The track will commence as a line, and almost immediately open into an ellipse, the lesser axis of which will extend as the larger axis diminishes, until it becomes a circle: what was before the lesser will then become the larger axis; and thus the motions will alternate until, from their decreasing magnitudes, they cease to be visible. . . . [See Figs. 51 and 54.]

In the most simple case of the coexistence of two sounds, shown by putting the entire rod into motion, and producing also a higher sound by the friction of a bow, the original figure will appear waved or indented, and as unity is to the number of indentations, so will the number of vibrations be to the number in the higher sound. On varying the mode of excitation, by striking the rod in different parts and with different forces, very complicated and beautiful curvilinear forms may be obtained; some of these are represented in Fig. 47. . . .

No. 2. Although very beautiful and varied forms may be produced from the motion of a single point, yet the compound figures which are presented by objects formed by a number of points offer appearances still more pleasing to the eye. . . . The mutual intersections of the points, each describing a similar figure, present to the eye complicated yet symmetrical figures, resembling elegant specimens of engine-turning.

When the plate is horizontal, the figures are all in one plane, but if it be inclined or perpendicular, the curves being then made in parallel planes, give the idea of a solid

figure, and in some cases the appearances are particularly striking. . . .

No. 3. When this prismatic rod is put in motion, in the direction of either of its sides, the points move only rectilineally; but when the motion is applied in an oblique direction, a variety of compound curves is shown. (Lissajous' figures). . . .

No. 4. When a rod is straight, the curve produced by any point describing its motion is always in the same plane;

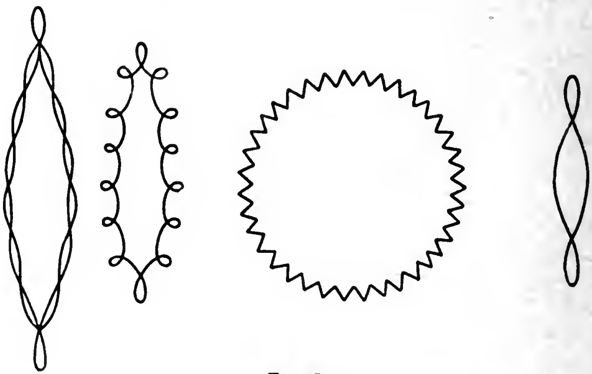

FIG. 47.

but in a rod bent to any angle, the two parts moving most frequently in different directions, curves are produced whose parts do not lie in the same plane. A few trials will soon indicate the best way of applying the motion so as to cause the two parts to vibrate in different directions."—Wheatstone, *Quarterly Journal of Science, Literature, and Art*, 1827, vol. i.

The above quotation has been given because the modern instruments are constructed to show only vibrations in one plane. Doubtless these are the most important, but the real Wheatstone Kaleidophone has been lost sight of.

Experiment LXXIX

Combination of Harmonic Motions (continued)

The instrument required, a double spring Kaleidophone, is shown in Fig. 48. It consists of two flexible strips of steel soldered together in the middle, the plane of the one being at right angles to that of the other. The upper strip has a bright metal bead fastened to it, which presents a small shining spot from whatever aspect it is viewed. Pull the lower strip aside: it executes a harmonic vibration at right angles to its own plane, and with it the upper strip; the bright spot then moves in a straight line, or, more correctly, an arc of a circle. So also if AB be drawn aside while BC is prevented from moving, it describes a straight line at right angles to the former. Now taking hold of the end A, pull it obliquely aside, and let it go, the spot then traces curves of great beauty, which go through various forms till through friction and resistance of the air the motion ceases.

Fig. 48.

Clamp the lower strip at different points and observe the change in the form of the figure. This is the advantage of the instrument as compared with the kaleidophone in which the lengths are constant.

Any wire clamped at one end and plucked aside goes through a similar series of motions, for there is always a plane of maximum and one of minimum flexural rigidity which work upon it in the way just described, but without some such device it gives only one figure.

Experiment LXXX

Blackburn's Pendulum

The apparatus shown in Fig. 40 can easily be converted into a compound or Blackburn's pendulum by sliding a small

ring, *r*, up the cord, so as to make it of a Y-shape (Fig. 49). The height of the bob will now require adjusting by means of the peg. The disc is drawn aside and fastened to an upright D by a thread which is burnt at the proper moment: this plan prevents any wobbling which might otherwise take

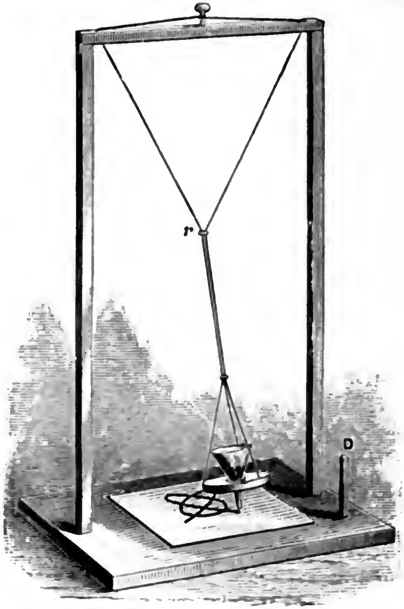

FIG. 49.

place. Sand is poured into the funnel as before, but it now describes a compound curve, derived from two harmonic motions at right angles. One of these, that parallel to the plane of the frame, is of the same amplitude and period as if the weight had been displaced from its position of rest in this direction, while the angle of the cord remained stationary in a vertical plane; the other, at

right angles to the frame, is executed by the entire length, reckoning from the middle of the under side of the cross-bar to the weight.

If we wish to obtain the curve combining harmonic motion of periods 1 : 2, we put the ring $\frac{3}{4}$ of the way down, counting from the top; and if the combination 2 : 3, at $\frac{5}{9}$, and so on.

The reason for this depends on the formula for a simple pendulum, $t = \pi\sqrt{\frac{l}{g}}$, where t = the time of a single oscillation in seconds, l = the length, and g the acceleration of gravity. The times are therefore directly as the square roots of the lengths, or what comes to the same thing, the number of vibrations in a given time varies inversely as the square roots of the length. For the 2 : 3 combination, then, the shorter portion must be to the total as 4 : 9, and reckoning from the top the ring r must be put $\frac{5}{9}$ of the way down.

Similarly, to obtain the ratio 3 : 4 we make the shorter portion $\frac{9}{16}$ of the whole, and put the ring at $\frac{7}{16}$ from the top.

If the paper, instead of being kept still, be moved with a uniform velocity in the direction of either of the component vibrations, we get the same curves as with the tuning-forks on pp. 115, 116. Lastly, when the paper itself moves with a harmonic vibration, as in certain "harmonographs," curves of exquisite beauty are obtained.

Experiment LXXXI

Lissajous' Method

The apparatus is shown in Fig. 50.

Two large tuning-forks are fixed to firm and heavy supports, so that their planes of vibration are at right angles. Each fork has a small piece of silvered glass at-

tached to one of its prongs in such a way that a narrow beam of light proceeding from a lamp is reflected first from one and then from the other. A small counterpoise is attached to the opposing prongs so as to maintain the balance. The observer stands at some little distance, and receives the twice-reflected beam through a telescope (the room

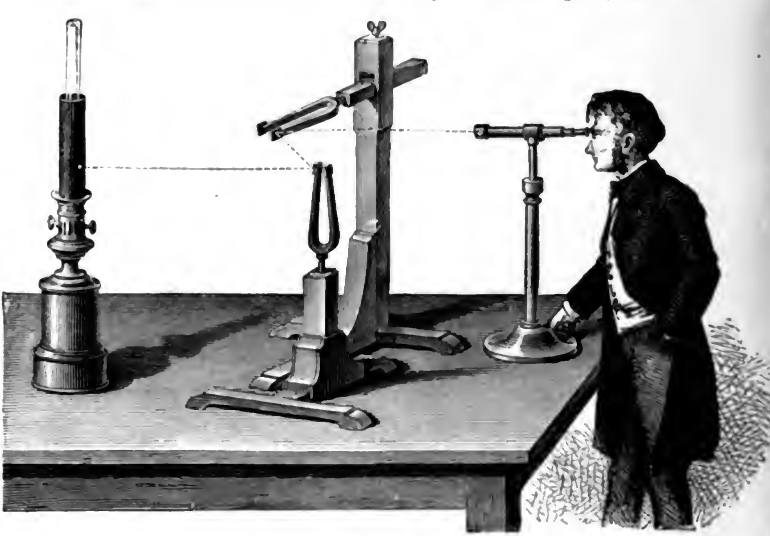

FIG. 50

being in darkness); when only one fork is vibrating he sees a straight line, vertical or horizontal as the case may be, but when both are in motion, curves such as those in Fig. 51 make their appearance, according to the rates of vibration. In general, they do not remain steady, unless the forks are perfectly in tune, but go through a cycle of changes in the period required for one of them to gain a complete vibration of the other, *i.e.* they go through all possible phase differences, as explained on p. 111. No other method of

tuning approaches this in delicacy. All the curves are comprised within a square or rectangle whose sides are twice the amplitudes of the vibrations, and in any of them the ratio can be determined by inspection. Count the number of times the curve touches two adjacent sides of

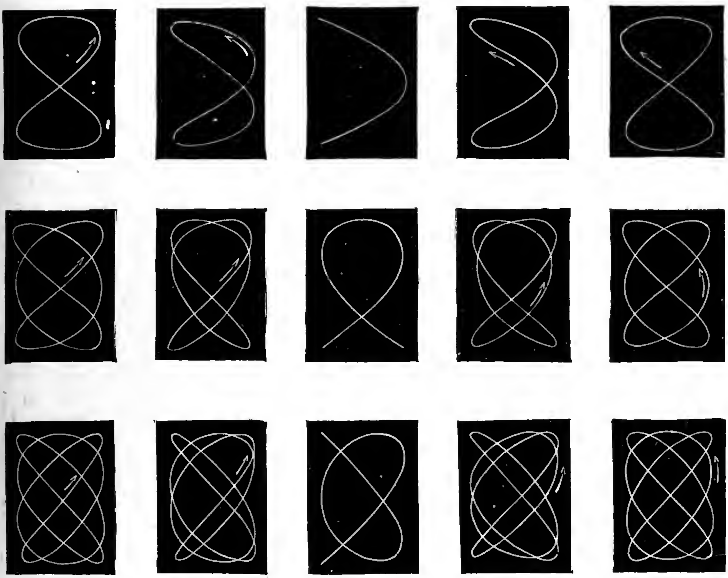

FIG. 51.

the rectangle which encloses it, and this is the ratio required. If the curve be not closed, as in the third *column* of Fig. 51, count one for each free end, and two for each of the other contacts. Thus, in the curve of the Fourth, 3 : 4 (middle figure, bottom row), we read 1 + 2 contacts along the top, and 2 + 2 down the right-hand side (or 1 + 2 + 1 down the left-hand side).

In practice the forks must be bowed from time to time by another operator, and some adjustment of weights will be necessary before the requisite steadiness is obtained.

Experiment LXXXII

Combination of Harmonic Motion (continued)

Required.—Apparatus shown in Fig. 52, consisting of two blackened discs of thin metal, each with a narrow diametrical slit cut out of it. These are fastened to the

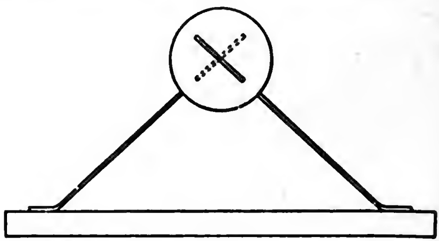

Fig. 52.

free ends of two steel strips, which are screwed on to the base board, and can execute harmonic vibrations at right angles to one another. The slits being also at right angles a small square of light is seen on looking through them at a bright surface beyond. When the steel strips are set into vibration this spot of light describes the compound vibration with which we are now familiar.

Graphical Representation—First Method

Take a sheet of thin transparent celluloid, about 35 × 12 cm. in dimensions, and trace upon it a single sine curve in Indian ink. To do this the template recom-

mended on p. 98 will be useful, but if there be not one ready, trace the curve first on paper, then lay it upon the celluloid, and prick holes in it at short distances to give the outline underneath; draw the curve in ink with a pointed camel-hair brush. Now bend the celluloid into a cylindrical form (Fig. 53), so that the two ends of the curve come together; the sheet will overlap, but that is of no consequence. Hold the cylinder vertically, and turn it round on a level with the eyes, then all the changes, from a straight line through an oblique and an upright ellipse to a straight line again, can be followed, as in Fig. 54. This method illustrates perfectly what is meant by *phase*; it is the angle through which the cylinder is turned, reckoned from one of the positions in which the curve appears as a straight line, or in the general case, goes through the centre of the rectangle. Now roll up the cylinder

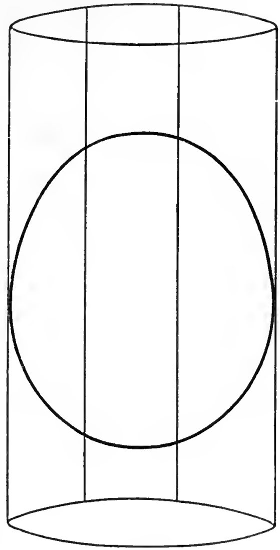

FIG. 53.

FIG. 54.

into a smaller compass, so that it makes two turns where it formerly made only one. Revolving it as before, we see

the octave combination: starting with a figure of 8, it becomes a distorted 8, then a parabola, and so on as in Fig. 51, 1st row.

To produce the combination 2 : 3 we must either draw two waves on the celluloid, and roll it up so that the first and last points do not meet till the circumference has been traversed three times, or draw three waves and roll it up into a double cylinder. The various phase differences show themselves as in Fig. 51, 2nd row. Similarly, any other combination might be produced by a proper adjustment of the number of waves and rolls.

The theory of this method is that uniform motion in a circle, when projected on a straight line in the plane of the circle, becomes simple harmonic motion; hence the projection of any single point on a revolving cylinder on a plane parallel to the axis describes a harmonic motion perpendicular to the axis. If the point at the same time describes on the cylinder itself a harmonic motion parallel to the axis, its projection will now be compounded of the two harmonic motions at right angles. On the celluloid cylinder all possible positions of such a point are shown, because the curve is continuous, hence the result.

Second Method

Draw two concentric circles (Fig. 55), whose radii are the amplitudes; divide them into the same number of parts (here 12), starting in both cases from the diameter AB, and number them as shown. Through each point in the outer circle draw a line perpendicular to AB, then the points where these lines meet the corresponding lines parallel to AB from the inner circle lie on the curve required. This is a well-known geometrical construction for describing an ellipse, whose axes are given.

If we continue the first horizontal and vertical lines to meet the diameters VI, XII, and 3, 9, we see that at

$\frac{1}{12}$th of the entire period from B to A and back the particle would be on the vertical axis by one movement, and

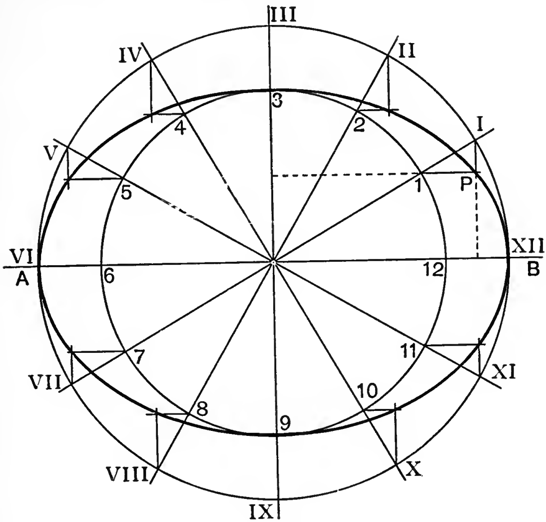

FIG. 55.

on the horizontal one by the other, hence its actual position is at P, where the dotted lines meet, and so on.

If we begin counting along the inner circle one place earlier, as in Fig. 56, and proceed as before, we obtain another ellipse which is tilted as compared with the last, and shows the effect of compounding two simple harmonic motions, whose phase difference is $\frac{1}{12}$th more than before, *i.e.* $90 + \frac{1}{12} \times 360 = 120°$. And so on for other differences.

Exercise. — Draw a single circle (to represent equal amplitudes in the two directions), and numbering the divisions one outside and one inside the circumference, find

what the phase difference must be so that the preceding construction gives a straight line.

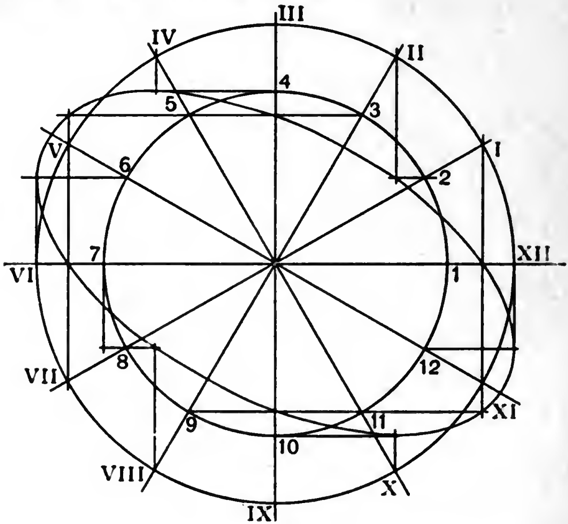

FIG. 56.

To compound harmonic vibrations of any other periods, say 4 : 5, draw one auxiliary circle ACBD (Fig. 57), thus making the amplitudes equal to avoid confusing the figure. Divide the semicircle ACB into $4n$, and DAC into $5n$ equal parts, here 8 and 10 respectively. Draw vertical lines through the former, and horizontal ones through the latter, continuing them in each case to the limits of the square. For a phase difference of $\frac{1}{4}$ of a period or 90°, start at one corner E, and continue diagonally till the curve comes to an end at H. All these diagonals (which are not of course straight) are described in equal times, and the varying length and curvature show to what extent the two motions are increasing or decreasing with

reference to one another. By starting at any other point than E, and following the diagonally opposite points all round, we get the same combination with a new phase difference, and the greater the number of parts into which the semicircles are divided, the more minute the possible

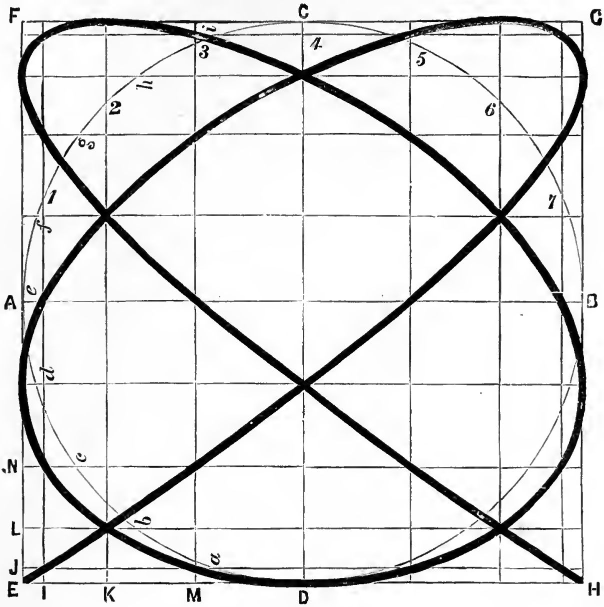

FIG. 57.

phase differences become. In Lissajous' experiment they go through *every* possible difference, unless the adjustment is perfect; then whatever figure is seen at first is maintained throughout, except that the size gradually diminishes.

Exercise.—Vary the preceding figure by drawing two auxiliary circles as in Fig. 56, but divide one into $3n$ and the other into $4n$ parts. Number them consecutively, starting

both sets at one corner. Draw the horizontal and vertical lines as before, and trace the curve through the points I 1, II 2, III 3, etc., till an open end is reached.

Also trace the effect of a new difference of phase by starting the numbering of one of the sets of parts one place later, then two places later, and so on, leaving the other unaltered.

Experiment LXXXIII

Combination of Two Parallel Harmonic Motions

Required.—Several large tuning-forks; soft wax; fine needle; oil flame.

Smoke one of the edges of a fork by holding it over a flame of oil, camphor, or turpentine. Strike the fork and draw the needle over the blackened edge; it traces a harmonic curve just as it would if attached to the fork, and drawn over blackened paper or glass. Now, instead of holding the needle in the hand, mount it on one of the other forks, so that it forms a continuation of a prong. Let both forks vibrate, and draw the needle as before. We thus get a compound curve traced, whose shape varies according to circumstances. In Fig. 58 are seen a number of such: the top line is the octave combination 2 : 1, the second line a slightly mistuned octave. Observe the gradual rise and fall of the small serration upon the large one, due to a constantly altering difference of phase. No. 3 is the combination 3 : 1, and the others are 3 : 2, 4 : 3, 5 : 4, 5 : 8, 25 : 24, and 81 : 80 respectively. All of them can be obtained by the method described, by taking the proper forks. If any ratio is not quite exact, a rise and fall is seen as in No. 2.

Since the general equation to a harmonic curve is $y = a\cos mx$, where a is the amplitude, and m a multiplier, the equation of those under consideration is $y = a\cos mx + b\cos nx$, where $m : n$ as $2 : 1$, etc., and a and b are the ampli-

tudes. If m and n are commensurable, the curve repeats itself after an interval, which in angular measure is the

FIG. 58.

least common multiple of $\frac{2\pi}{m}$ and $\frac{2\pi}{n}$, for then the value of y is the same as at first.

Experiment LXXXIV

Composition of Two Rectangular Harmonic Motions

Apparatus as before. Instead of drawing the needle longitudinally down a blackened prong, draw it trans-

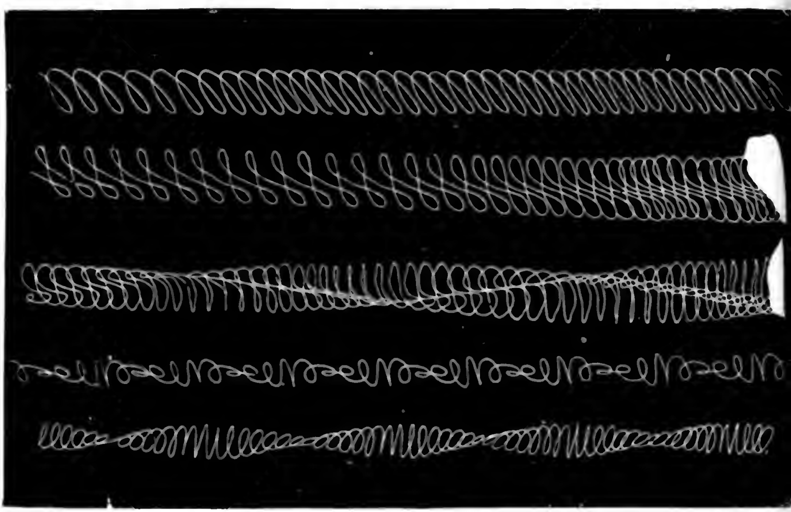

Fig. 59.

versely, *i.e.* at right angles to its length. It is better to fasten a strip of blackened glass, copper foil, or mica on the prong, so that the drawback of a narrow edge may be avoided. The curves now obtained resemble those in Fig. 59, which are compounded of periods 1 : 1, 2 : 1, 2 : 1 slightly mistuned, 6 : 5, and 16 : 15 respectively.

The form of the curve varies somewhat with the speed of the needle; it forms a very interesting experiment to try various speeds and combinations of periods. The curves are the same as those in Fig. 51, compounded with a uniform rectilinear motion.

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CHAPTER X

REFLECTION AND REFRACTION OF SOUND

Experiment LXXXV

Required.—Two long, wide glass tubes; piece of card; a watch.

Arrange as in Fig. 60, the watch being at A, the card at C, and the ear at B. If the ticks can be heard across

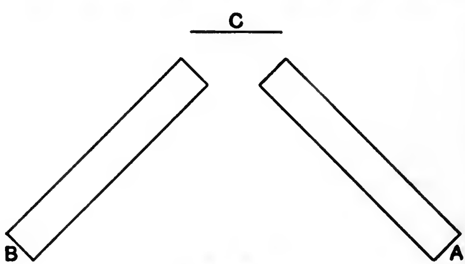


Fig. 60.

the interval, cover the watch with a piece of cloth, which also encloses a little of the tube. It is possible not only to prove the fact of reflection in this way, but to show that the law of the equality of the angles of incidence and reflection is at least suggested, for by turning the card it will be found that the ticks are loudest when it is placed symmetrically. Other surfaces, hard and soft, may be

substituted for the card, and the reflection compared. Even the flat side of a fish-tail flame is efficacious.

The principle of reflection may be explained as follows. Let the source of sound be at S (Fig. 61), and let there be a reflecting surface AB some little distance away. A system of spherical waves will emanate from S, and on striking AB, the first wave-front, instead of reaching its

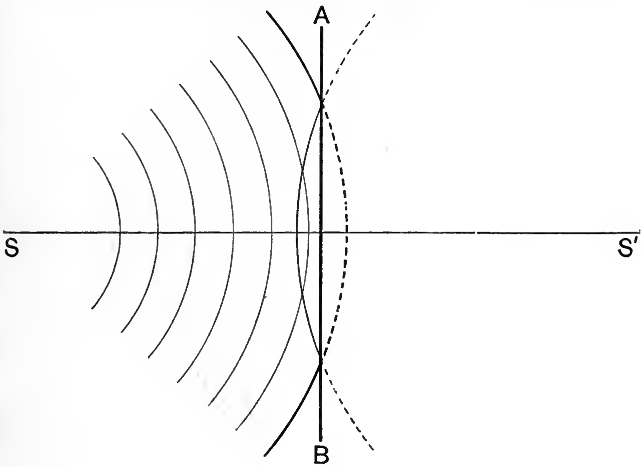


FIG. 61.

natural limit, as shown by the dotted line, will be turned back. It will go on expanding, and be followed by other waves; and since the curvature is the same, but in a contrary direction, the source will appear to be at S′, which is as far behind the reflector as S is in front of it. An echo, therefore, is heard after the interval required for the sound to travel from S′, supposing that it started at the same moment as the original sound. This of course amounts to nothing more than saying that the sound has to traverse the distance from S to AB twice

over before it can be heard at its starting-point. A reflected sound is not called an echo unless it is heard at some perceptible interval after the original, otherwise it comes under the head of Resonance. The meeting of direct and reflected pulses gives rise to stationary waves, as explained on p. 11 and elsewhere.

Very striking examples of reflection are observed in a large room with bare walls, such as a covered fives court, where the difficulty of making one's self understood at a distance is very great.

EXPERIMENT LXXXVI

When a concave mirror is used the reflected wave-fronts take up the additional curvature, and are rendered convergent; from a convex surface they become more divergent. Take for instance a source of sound at A (Fig. 62), and let a concave reflector BC be put in front of it, then all the portions of waves comprised within the angle BAC will after reflection be confined to some such angle as BA′C, and the intensity at the point A′ will be a maximum. With a convex reflector the lines A′B and A′C would diverge after reflection. These considerations belong rather to Optics than Acoustics, for the excessive minuteness of light waves enables them to be treated as mathematical lines, and the effects are beyond comparison more exact than with sound waves, which usually have a length comparable with the size of the reflector. In such cases a flat surface does nearly as well as a curved one.

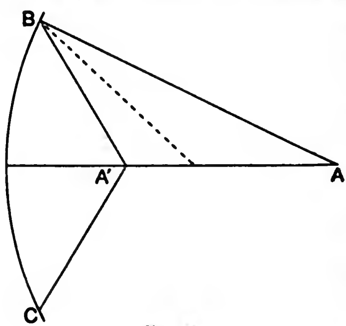


FIG. 62.

Experiment LXXXVII

Reflection from Curved Mirrors

Required.—Two concave metal mirrors, stands, watch, rubber tube, and small funnel.

Set up the mirrors as in Fig. 63, the distance between them being 4 or 5 metres. Hang the watch about half-way between one of the mirrors and its centre of curvature. Hold the funnel, with tube attached, in a corresponding position near the other mirror, and move it about till the ticks are plainly heard.

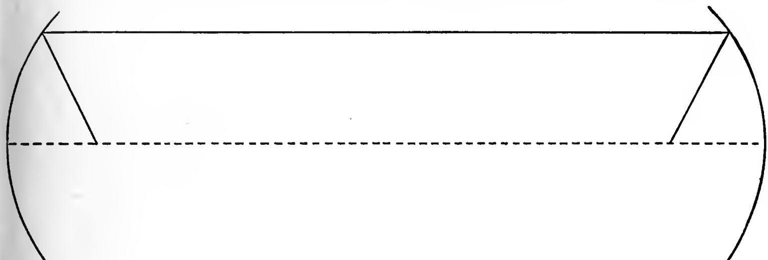


Fig. 63.

A single mirror will suffice to show the concentration of sound waves to a focus, as in Expt. LXXXVI., but the watch should now be a couple of metres away, and the funnel turned towards the mirror. An open umbrella may be used with very satisfactory results.

As another instance, hang a watch on a vertical sheet of cloth or a curtain, to prevent reflections from behind, and hold a wash-basin near the ear in such a position as to catch the waves and condense them to a point: the ticks can be heard across an ordinary room. A very slight change in the position of the ear or basin destroys the effect altogether.

EXPERIMENT LXXXVIII

Reflection of Surface Waves

Required.—Shallow circular dish containing mercury.

Agitate the surface at any point, and observe how the waves after reflection meet at a corresponding point equally distant from the centre. The agitation is best kept up by allowing a thin stream of the liquid to trickle down from a tap-funnel or filter-paper with the apex cut away as in Expt. CX., p. 163. The experiment also succeeds with water, but not quite so well.

It was pointed out many years ago by Scott Russell that when water waves strike a bank or other solid surface at a small angle, they are not reflected at all, but cling to and run along it. Something of this kind probably happens in whispering galleries, where the speaker puts his mouth so close to the wall as to forbid the idea that the ordinary laws of reflection are concerned.

Reflection from Railings

Stand a short distance from a set of iron railings with square sides : clap the hands together, or rap on the ground with a stick ; a little attention will reveal a musical clink of some duration. The loudest effect is produced when the bars are arranged in a crescent, and have a wooden backing close behind, or when wooden strips are nailed together in a zigzag fashion. It is due to reflections from the successive surfaces, reaching the ear with sufficient rapidity to blend together. A single wave thus comes back, as it were, in pieces from gradually increasing distances. With a little practice the sound may be heard with almost all kinds of railings, but a very small difference in the situation of the observer has a considerable influence

over the result. A high-pitched tiled roof offers another example: traffic in the street below produces sounds like the twittering of birds, coming now from one spot, now from another. Here the narrow edges of the tiles are responsible for the effect.

Experiment LXXXIX

Refraction of Sound

Required. — Collodion balloon, retort stand, carbonic acid generator, with thistle-funnel and delivery tube.

Put in the bottle some fragments of marble, pour dilute hydrochloric or nitric acid on them through the funnel, and the gas comes off readily. Fill the balloon, and tie the neck with a piece of thread. Then hang it from a retort stand, and keep it steady by letting the lower part rest on one of the rings. Suspend a watch at a distance of 15 or 20 cm. on a level with the centre of the balloon, and place the ear on the opposite side. Find by trial the position where the ticks are loudest. It will be abundantly clear that the balloon does not act as an obstacle, shutting off the sound, but as a lens, converging it to a focus. This experiment was devised by Sondhauss.

The theory of it is that sound travels with a less velocity in a heavy gas than in a light one; consequently the wave-fronts, as shown in Fig. 64, become flatter as they pass through the gas and then take a contrary flexure, from the circumferential parts gaining on the central ones. On emergence, the new flexure is accentuated, and at a point whose distance varies with that of the watch on the other side, the waves converge to a focus.

In passing from air to water, sound is almost totally reflected, even at a perpendicular incidence, on account of the great difference in density. Hence it is difficult,

though not impossible, for a diver to hear sounds made in the air above his head. Lobsters, fishes, etc., are startled

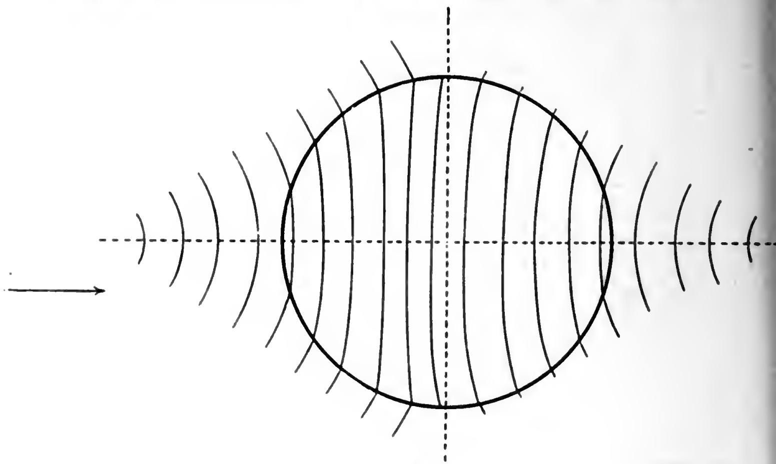


FIG. 64.

by the firing of guns and other loud noises made in the air, under circumstances where it seems impossible that any sound can reach them except by refraction.

EXPERIMENT XC

To illustrate the preceding by the Analogy of Water Waves

Take a large flat vessel, a common portable bath for example, fill it about half full of water, and place a flat disc of wood or glass in a horizontal position on a support, so that it rests just underneath the surface of the water. Make a succession of waves in the water at some convenient point, by blowing bubbles of air through a fine glass tube. Watch the change and reversal of curvature

as the waves pass over the disc. The effect here is due to the velocity of a wave in shallow water being less than in deep water: hence we have an exactly similar case to that described above. A lens-shaped disc would doubtless do better than a circular one.

Experiment XCI

Refraction through a Hydrogen Balloon

Use the same balloon as in Expt. LXXXIX., but fill it with hydrogen instead of carbonic acid. No focus can now be detected, and the sound is perceptibly weaker when the balloon is there than when it is removed.

For hydrogen being nearly 15 times lighter than air, the velocity of sound in it is $\sqrt{15}$ or nearly 4 times greater: hence the wave-fronts, in passing through the balloon, will become more convex than they were, and on emergence the difference will be intensified.

It may seem curious that a convex lens can cause rays to diverge, whereas in the case of light this is never looked for. But the conditions are not the same if the rays pass from air to glass, and then to air again. If we sink an empty carbon disulphide prism into a glass trough filled with water, and pass a beam of light through it, it is bent *away* from the base: similarly, if we use a hollow lens the rays are *diverged*. But a little consideration will show that the proper shape for a lens to have under these circumstances in order to produce a convergence, is concave. So also a concave lens, filled with hydrogen, would converge sound waves to a focus in air.

Tyndall's experiments on the audibility of guns, whistles, and sirens in different states of weather may be referred to here, the principles involved being that in a homogeneous atmosphere a sound is heard at a distance proportionate to its intensity and varying with its quality; but

in one where there are strata of different temperature or humidity, reflections and refractions occur which have a remarkable deadening effect, and are capable of extinguishing a very loud sound at a distance of two or three miles, even while the instruments are in full view.[1] On a small scale the cutting off of a sound by a flame (Expt. IV.) and its reflection by the same means, sufficiently indicate the principle.

Heschus[2] has investigated the refraction of sound through a large plano-convex lens 25 cm. in diameter, made of wire netting, and filled with fragments of sponge, ebonite shavings, wadding, etc., using as the source of sound a Galton's whistle (p. 192). In this case the sound waves are hampered in passing through the materials, and a change of flexure and convergence to a focus take place as indicated above.

With regard to prisms, some experiments were made by Hajech[3] on the relation between the angles of incidence and refraction, with the following results:—

Material.	Angle of Incidence.	Angle of Refraction (obs.).	Calculated.
Water	35°50′	7°40′	7°58′
,,	25°	5°40′	5°37′
Hydrogen	35°50′	8°	8°50′
,,	25°0′	7°	6°22′
Ammonia gas	41°0′	29°20′	30°22′
,,	35°50′	25°	26°50′
Carbonic acid	35°50′	49°50′	48°19′
,,	25°	33°20′	32°33′
Sulphur dioxide	35°50′	62°30′	61°22′
,,	25°	40°	39°24′

The calculated values were obtained from the principle that sine of angle of incidence : sine of angle of refraction = velocity of sound in air : velocity in the gas = $\sqrt{}$(density of gas) : $\sqrt{}$(density of air).

[1] Tyndall, *Sound*, chap. vii. [2] *Phil. Mag.* 1892.
[3] *Annales de Chimie et de Physique*, 1859.

CHAPTER XI

VELOCITY OF SOUND

THE velocity of a longitudinal wave in any substance is, in the general case, given by Newton's formula $v = \sqrt{\frac{E}{\Delta}}$, where E is the elasticity—Young's modulus for solids, and the ratio of a small increment of pressure to the (relative) decrement of volume thereby produced for liquids and gases)—and Δ the density. By Young's modulus is meant the ratio of a small increment of pressure to the resulting diminution of length.

In gases the elasticity is numerically equal to the pressure, for if P and V be the pressure and volume at any temperature, then a small increase p in the pressure will produce a corresponding decrease v in the volume.

Now, by Boyle's law, $(P+p)(V-v)=PV$, or $\frac{P+p}{P}=\frac{V}{V-v}$, and by division $1+\frac{p}{P}=1+\frac{v}{V}+\left(\frac{v}{V}\right)^2+$ etc., whence $\frac{p}{P}=\frac{v}{V}$ (neglecting squares and higher powers, since v is very small compared with V) and $P=\frac{pV}{v}$. But by the definition $p \div \frac{v}{V}$ is the elasticity and therefore = the pressure.

The length of a column of gas of uniform density, whose weight is equal to this pressure, corresponds to what has been called the "tension-length" in the case of a string

(p. 27). When air is in question, this is called the height of the homogeneous atmosphere. The velocity of sound in any gas is (apart from thermal considerations) equal to that of a body after falling freely under gravity from a height = half the length of this column. Take the case of air, the normal pressure upon it is $76 \times 13{\cdot}596 \times 981 =$ $1013663{\cdot}376$ dynes to the sq. cm., and by supplying the value of Δ($\cdot001293$ grs. per cc.) we get the velocity of sound $= \sqrt{\dfrac{1013663{\cdot}376}{\cdot001293}} = 28358$ cm. per second.

This is Newton's value, and is about $\frac{1}{6}$ less than the true one, as was known even in his day. The discrepancy was explained by Laplace, who showed that the heat developed in the condensed portion, as well as the cold in the rarefied portion, would both concur in augmenting the velocity, and, in fact, that the elasticity should be reckoned as γE, where γ is the ratio of the specific heat of air at constant pressure to that at constant volume.

For air and elementary gases of low atomic weight, the value of γ is about $1{\cdot}41$, the square root of which is $1{\cdot}187$. Multiplying by this quantity we get $v = 33231$ cm. per second, which agrees with the best experimental determinations within the limits of error.

The rate of exchange of temperature between the condensed and rarefied pulses depends on the nature of the gas, being greater with those of high radiative power, such as coal-gas, sulphurous acid, ammonia, ether vapour, etc., and less with hydrogen, oxygen, nitrogen, etc. In fact, the correction applies in its full value only to the latter class, while in the former it becomes very small. With a very slow frequency the equalisation of temperature would be nearly complete; with a very high one it would not be so: hence Newton's formula is approximately fulfilled in the one case, and Laplace's in the other, independently of radiative power.

The velocity is not independent of amplitude: a very loud sound, on theoretical grounds, should travel faster than one of moderate intensity,[1] and in practice this is found to be the case.[2]

As regards temperature, the resistance of a gas to compression increases with a rise, and diminishes with a fall; the velocity being expressed by the formula $v = \sqrt{\frac{\gamma E}{\Delta}(1 + at)}$, where a is the coefficient of expansion for 1°, (·003665), and t the number of degrees. When calculated out, the increase amounts to between 60 and 61 cm. per second for each degree centigrade.

A change in the pressure alone has no effect, for it alters both E and Δ in the same proportion. A change in humidity alters Δ because moist air is lighter than dry air. The proportion is very small, about 1 part in 200 or 300, according to circumstances. The exact amount may be calculated from tables.

In solids the elasticity, and also the velocity of sound, become smaller for a rise of temperature. In liquids they increase. The correction for specific heats is still applicable on the same grounds as before, but is so small as to be negligible.

In practice the methods for finding the velocity of sound in all materials usually resolve themselves into a determina-

[1] See Rev. S. Earnshaw, *Phil. Mag.* 1860. Also W. W. Jacques, *Phil. Mag.* 1879, "On the Velocity of Loud Sounds."

[2] The most interesting example is that recorded in one of Captain Parry's Arctic voyages. The Rev. John Fisher, astronomer to the expedition, states that "on the 9th February 1822 . . . the officer's word of command to 'fire' was several times distinctly heard, both by Captain Parry and myself, about one beat of the chronometer ($\frac{1}{2}$ sec.) after the report of the gun. The word 'fire' was never heard during any of the other experiments. Upon this occasion the night was calm and clear, the thermometer 25° below zero (Fahrenheit), and the barometer 28·84 inches, which was lower than had ever been observed before at Winter Island." The distance was about $2\frac{1}{2}$ miles. No doubt can be thrown on the accuracy of this observation, but it has hitherto remained an isolated one.

tion of wave-length and the corresponding frequency, but these experiments only give its value in a wire, rod, or tube, while theory gives it in the unlimited medium, but it is possible to allow for this. Kirchhoff obtained the following formula. Let $v =$ observed velocity in a tube filled with gas, V the velocity required, $2r$ the diameter of the tube, and n the frequency: then $v = V\left(1 - \frac{c}{2r\sqrt{\pi n}}\right)$, where c is a constant for friction and conduction of heat. Using two tubes whose radii are r_1 and r_2, and the same frequency, we obtain

$$V = \frac{v_1 r_1 - v_2 r_2}{r_1 - r_2}.$$

The theoretical value of c is ·00742: by experiment it is ·008.[1]

Exercise.—Test the above formula, using the same fork, and tubes of different diameters, and making a correction for temperature, as explained above.

Velocity in Air

Of direct methods the following, due to Bichat (*Nature*, November 28, 1878), may be described. An iron tube 10 m. long is bent so that the branches lie side by side, and one end is closed by a stretched sheet of indiarubber. An opening in the tube near this is connected with a manometric capsule (see p. 186), the other end is closed by a stopper also joined up to a capsule, and both of these have styles attached, so that when agitated they make marks on a blackened revolving cylinder. A wave is started by tapping the sheet of rubber, and the time interval between the marks is found by comparison with the wavy curve traced by a tuning-fork on the same cylinder.

Another method, devised by Bosscha, consists in causing

[1] J. W. Low, *Phil. Mag.* Sept. 1894.

two sets of simultaneous clicks to be made at the rate of 10 per second by an electro-magnetic contrivance, one set being near the ear, and the other at a distance which can be varied. The latter is moved away till the clicks from it arrive exactly one interval late: when this happens, the sound traverses a known distance in a known time, hence the velocity can be calculated.[1]

Experiment XCII

Velocity of Sound in Air. I

Apparatus as for Expt. XXIX. p. 35.

Find the length of maximum resonance as before, add ·8 of the radius of the tube, and multiply by the frequency. Take the temperature in the tube, multiply by 61 to get the number of cm. per second, and subtract to get the velocity at 0°.

Example.—A tube 15·3 cm. long by 2·6 cm. diameter resounded to $c'' = 512$ v.s., temperature 16·0° C.

The calculated velocity at this temperature is $512 \times 4 \times (15{\cdot}3 + {\cdot}8 \times 1{\cdot}3) = 33463$ cm. per second. At 0° it would be 16×61 cm. per second less, or 32487.

Experiment XCIII

Velocity of Sound in Air. II

Required.—Tube of glass or metal about a metre long and 3 cm. wide. Also an outer tube or cylinder stopped at one end, and wide enough to contain the preceding, but its length need not exceed $\frac{3}{4}$ m. This tube is nearly filled with water. Tuning-forks and clamps to hold the tubes.

In this experiment, find first the shortest length of column which will resound to each fork, and then the next

[1] *Pogg. Ann.* vol. xcii. 1854.

longest. The difference between the measures is half a wave-length for that particular note, because there is a node at the water-level in each case. The following are some results obtained :—

Frequency of forks	256,	320,	384,	445,	512.
Distance from end of tube to 1st node	31·6,	27·4,	20·5,	17·3,	15·0 cm.
,, ,, 2nd ,,	97·8,	80·4,	64·8,	55·2,	47·9 ,,
Differences (= ½ wave-length)	66·2,	53·0,	44·3,	37·9,	32·9 ,,

Multiplying by twice the frequency in each case, we get the values of the velocity 338·9, 339·2, 339·4, 337·3, 336·9 m. per second.

Mean = 338·6 ; temperature 16·4°. Reducing to 0° by subtracting ·6 × 16·4 m. it becomes 328·8 m. per second.

A very fair degree of accuracy may be obtained by this method when every precaution is taken to ensure success, *e.g.* rating the forks and making a temperature correction for them, allowing for humidity as well as temperature in the air itself, and making a large number of experiments.[1]

A still longer tube gives other positions of maximum resonance as the successive nodes are reached, but they gradually become less well defined.[2]

Experiment XCIV

Velocity in other Gases

Required.—Glass tube dipping into water, as in Expt. XXIX. ; delivery tube reaching nearly to the bottom, and supplying the gas experimented on (which in this case must be heavier than air) ; tuning-fork.

The mode of conducting the experiment is the same as that already described. A gentle current must be kept up,

[1] See a paper by D. J. Blaikley, *Phil. Mag.* May 1879.
[2] Mayer, *Phil. Mag.* March 1894.

and the position of maximum resonance found as usual. Carbonic acid is most convenient for the purpose.

A more accurate determination can be made by also obtaining the second node and measuring the distance between them, as in Expt. XCIII.: in this way the effect of diffusion can be avoided.

When the gas is lighter than air, *e.g.* coal-gas, a tube closed by a movable plug must be used, and held in an inverted position. The correction to be added for the diameter of the tube is applicable in all cases.

The values obtained may be compared with the calculated ones, derived from the formula $v = V\sqrt{\frac{d}{d'}}$ where $v =$ velocity required, $V =$ velocity in air, $d' =$ density of gas, and $d =$ density of air (= ·0013 grs. per cubic cm.). The ratio γ has a different value for each gas, so that the formula gives only an approximately correct value. The following are some experimental results :—

Carbonic acid	257·3	m.	per second.
Hydrogen	1237·6	,,	,,
Ether vapour	175·8	,,	,,

Experiment XCV

Kundt's Method

Required.—Apparatus shown in Fig. 65. BB′ is a glass tube, $1\frac{1}{2}$ m. long by 3 cm. diameter (a much smaller one may be used, but the effects are then less easy to obtain). It must be quite clean and dry inside, and at one end it is closed by a cork *b*, fitting somewhat tightly, but capable of adjustment by a wire handle. At the other end a bored stopper KK is placed, which admits a brass or glass tube AA′, about 1 m. by 1 cm. The latter is surrounded for nearly half its length by the outer tube, and carries a disc *a*, which nearly touches it all round. Both tubes, but

especially the outer, require to be held in position by clamps. To make an experiment a small quantity of lycopodium or dried precipitated silica is introduced into the space aB and made to lie evenly along the bottom. Now on stroking the inner tube with a resined glove or a piece of wet cloth the powder gathers into heaps at regular intervals as shown. If it does not do so at first, alter the position of the plug b. Measure the distances between the heaps and take the mean of several experiments. With an inner tube of the length taken this is about 62 mm. If we know the frequency of the tube AA′ vibrating longitudinally, we have only to multiply twice the distance between successive heaps by this number to obtain the velocity required.

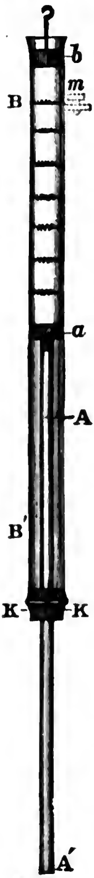


FIG. 65.

The theory of the method is that by the to-and-fro motion of the disc a series of stationary waves is set up between a and b, just as when a long indiarubber tube is attached to a support at one end and agitated at the other. The powder collects at the nodes, so that the distance between any heap and the next is half a wave.

Since the air in ab vibrates in the same manner as the material of the tube AA′ (*i.e.* longitudinally), and the rapidity of vibration of each is the same, the half-wave in air (or distance between two successive heaps) is to the half-wave in AA′ (the length of the tube) as the velocity of sound in air is to that in the material of this tube. Hence we might find the velocity in brass, copper, glass, etc., without any difficulty, but unfortunately the method, so beautiful in theory, fails to give accurate results (for solids) in practice. It is found that in no case are the heaps equally spaced, nor is the pitch of the sounding tube quite

uniform. When the length of the tube is not an integral multiple of a quarter wave-length the dust lies in transverse striæ, because the particles experience a force tending to separate them longitudinally, but to unite them transversely. When gripped by sheets of indiarubber covered with silk the joint is not so rigid as with a cork or stopper, and yet is air-tight. By filling the outer tube with coal-gas, hydrogen, carbonic acid, etc., and again finding the distances between the heaps, the velocity in these gases may be found: it is of course proportional to the wave-length when the frequency is the same. In some of Kundt's experiments there were two outer tubes, one at each end, containing air and the other gas respectively. The sounding tube was made to vibrate in its second mode (with two nodal points), and thus both gases were under identical conditions.

This is the general method for finding the velocity of sound in a gas, air being taken as the standard, and upon it is based the determination of the quantity γ, which gives a clue as to whether a gas like argon, say, contains two atoms to the molecule or only one; in the former case work is done upon the molecule itself, and a proportionate quantity of heat disappears in molecular energy; in the latter there is no such absorption. The inner tube is not really necessary, for introduce lycopodium all along the tube BB′, plug it at both ends and clamp it in the middle, then make it vibrate by stroking it, and the powder lies in distinct forms which mark out so many half-waves.

The formula $v = \sqrt{\frac{\gamma E}{\Delta}}$ gives $\gamma = \frac{v^2 \Delta}{E}$, where Δ is the density of the gas (in grams per cubic cm.), v the velocity in cms. per second calculated from the observation, and E the elasticity, which we have seen is equal to the pressure, viz. $76 \times 13{\cdot}596 \times 981 = 1013663{\cdot}376$ dynes per square cm. under the standard conditions.

Taking air as an example, we have at 0°, $v = 33200$ and $\Delta = \cdot 0013$. Hence

$$\gamma = \frac{33200^2 \times \cdot 0013}{1013663} = 1 \cdot 413.$$

The limiting value of γ is unity, and the more it departs from this the less rapid is the equalisation of temperature between a condensed and a rarefied pulse.

Experiment XCVI

Velocity of Sound in a Wire or Rod

Required.—Sonometer; resined rag; bow; etc.

Tune one wire, which is to be used as a standard, to a known frequency, say $c'' = 512$, measure its length (L), rub the experimental wire longitudinally: it will give a note of high pitch. Slide the movable bridge along the other wire till the shorter portion (vibrating transversely) gives out this same note; let its length be l. Then, since the number of vibrations is inversely as the length, we have, where n is the number required, $\frac{n}{512} = \frac{L}{l}$. This is also the frequency of the longitudinal vibration, and, since the wire vibrates in its simplest mode, it comprises half a wave-length. The velocity is now found by multiplication.

Example.—A brass wire 1 m. long rubbed longitudinally, gave the same note as 3·1 cm. of a wire, which in its whole length, 99·6 cm., gave 512 v.s. vibrating transversely.

The frequency corresponding to 3·1 cm. is $\frac{512 \times 99 \cdot 6}{3 \cdot 1} =$ 1645, and the velocity in brass wire $= 1645 \times 2$ m. $=$ 3290 metres per second. When a more accurate determination is desired, a special form of sonometer is used, longer than the ordinary one, and having massive clamps at each end. The pitch is independent of the tension.

The velocity in tubes of glass, copper, brass, etc., may

be found in a similar way, holding them as shown in Fig. 66.

By calculation from the formula $v=\sqrt{\frac{E}{\Delta}}$ we may check the experimental results. The following are some values of E, Δ, and v:—

	E	Δ	v
Glass .	$5{\cdot}74$ to $6{\cdot}03 \times 10^{11}$	2·5 to 2·7	4800 m. per second.
Brass .	$9{\cdot}48$,, $11{\cdot}2 \times 10^{11}$	8·5	3420 ,,
Copper .	$11{\cdot}72$,, $12{\cdot}34 \times 10^{11}$	8·3 to 8·9	3730 ,,
Steel .	$20{\cdot}2$,, $24{\cdot}5 \times 10^{11}$	7·8	5340 ,,

It must be observed that the formula gives the velocity

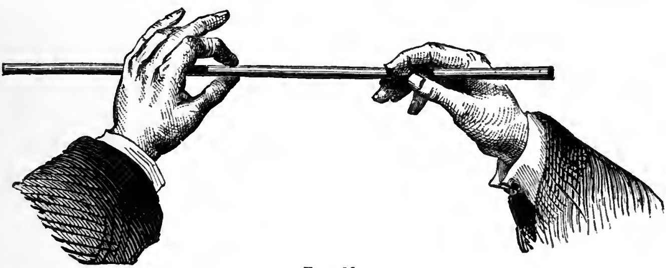


Fig. 66.

in an unlimited medium, whereas experiment gives it in a wire or tube. The latter value should be multiplied by $\sqrt{\frac{3}{2}}$ or 1·224 to bring it up to the calculated one.

Exercise.—Find the ratio of the velocities of longitudinal and transverse vibrations in a given wire.

According to theory[1] the velocity of transverse vibrations is $v_t=\sqrt{\frac{T}{m}}$, and of longitudinal, $v_l=\sqrt{\frac{E}{\Delta}}$; hence $\frac{v_l}{v_t}=\sqrt{\frac{E}{\Delta}\times\frac{m}{T}}$, but $m=\pi r^2\Delta$; so that $\frac{v_l}{v_t}=\sqrt{\frac{E\pi r^2}{T}}=r\sqrt{\frac{E\pi}{T}}$ where E and T are both measured in dynes. If we put

[1] See p. 27.

$F = \frac{T}{\pi r^2}$ so that F is the stretching force per unit of area, then $\frac{v_l}{v_t} = \sqrt{\frac{E}{F}}$. This result should be tested. As a force equal to E would instantly snap any ordinary wire to which it was applied, while T is generally well under the breaking limit, the velocity and hence the frequency in a wire vibrating longitudinally is much higher than when vibrating transversely.

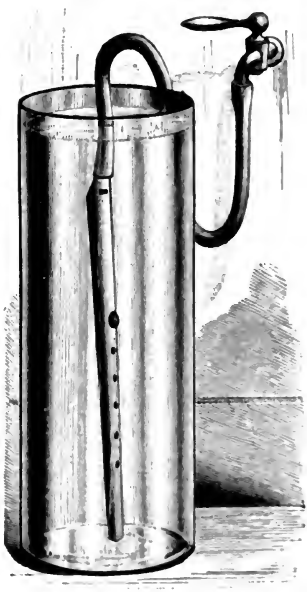


Fig. 67.

Velocity in Liquids.

The same formula $v = \sqrt{\frac{E}{\Delta}}$ enables us to calculate the velocity of longitudinal waves in liquids : some values of v are given in the following table :—

Water	1431	m. per second at	8°
Alcohol	1172	,,	23°
Ether	1155	,,	12°

Direct methods are only possible in the case of water, but there are two indirect ones which are available. One is to force the liquid under consideration through a metal flageolet, as indicated in Fig. 67, and obtain the frequency by any of the usual methods. The wave-length is twice the distance from the lip to the first *open* hole. In the figure the holes are all open except the first.

Another method is to fill a tube such as is used for Kundt's experiments entirely full of water or other liquid,

and to introduce some finely divided iron (obtained by reduction from the oxide) instead of lycopodium or silica. The apparatus is described in *Nature*, June 3, 1875: it appears that the thickness of the walls of the tube has an influence on the result. With water a velocity of 1040·4 to 1382 m. per second was calculated. A direct determination in the Lake of Geneva by Colladon and Sturm (1827) gave 1412 m. per second at 0°.

Melde (*Phil. Mag.* Feb. 1892) has obtained the velocity in membranous substances by finding what note a certain length of each gave when rubbed longitudinally. The following values are quoted from his paper. Paper soaked with wax, 3040 m. per second, satin ribbon 1950, string 1720, cotton 1280.

CHAPTER XII

DÖPPLER'S PRINCIPLE, AND INTENSITY OF SOUND

WHEN a source of sound approaches or recedes from the ear, more or fewer waves are caused to meet it than if it were stationary, and when the motion is sufficiently rapid an appreciable rise or fall in pitch is observable. This is known as Döppler's principle, and in Optics it has been applied with great success in determining the motion of the stars towards or away from the earth, by finding the extent to which lines in the spectrum are displaced towards the violet or red end, and calculating what amount of mutual approach or recession would bring about this change of refrangibility. In the case of sound the phenomenon presents itself when a locomotive blowing a whistle rushes past an observer, who is apt to fancy that the handle of the regulator has been turned first one way and then the other.

EXPERIMENT XCVII

Required.—Small organ pipe; rubber tubing.

Let one observer take the tube, which is about 2 m. long, and having attached the pipe to one end, let him swing it in a horizontal circle above his head, while he blows through it at the same time. To another observer

in a different part of the room the pitch rises and falls according to which side of the circle the pipe is on, approaching or receding.

The velocity of rotation may be calculated roughly from the difference in pitch. Thus suppose the pipe has a frequency of 1024, and that it appears a chromatic semitone sharper on the one side than the other. This means that the fluctuation is nearly 30 v.s. on each side. Taking the velocity of sound as 33800 cm. per second, the alteration of pitch (see below) is $\frac{33800+a'}{33800-a'}=1{\cdot}059$ (a chromatic semitone), from which $a'=9{\cdot}7$ m. per second. If the radius is $1\frac{1}{2}$ m. the circumference is 3π or 9·4 m., and dividing by 9·7 we get 1 second, nearly, as the time of a revolution. This may be compared with the observed time as taken by a watch.

The complete investigation of Döppler's principle, taking into account all possible varieties of motion, is as follows.[1]

Let a, a', and m be the velocities of the observer, source and medium respectively, all resolved in a direction from the source to the observer (evidently displacements in a direction at right angles to this would not affect the result). Let n be the number of vibrations per second performed by the source and v the velocity of sound in the medium.

Then $v+m=$ the actual velocity of sound (*i.e.* relatively to the earth), and $v+m-a'$ is its velocity relative to the source. Hence the wave-length is $\frac{v+m-a'}{n}$. Its velocity relative to the observer is $v+m-a$, hence the number of waves which reaches him per second is $\frac{v+m-a}{v+m-a'}\times n$, this being the velocity relative to the observer divided by the wave-length. In general the source only is in motion, so that m and a both $=0$. Then if n have the same meaning as

[1] Deschanel's *Physics*, by Everett.

before, $\frac{v \pm a'}{n}$ is the wave-length, and v is the velocity of sound relative to the observer; so that $v \div \frac{v \pm a'}{n}$, or $n \times \frac{v}{v \pm a'}$, is the effect of the motion, according as it is from or to him.

Intensity

We can only consider this subject theoretically, as it does not lend itself to experimental treatment, especially on a laboratory scale. The ear is not capable of estimating the relative loudness of two sounds with any accuracy, even if they are of the same pitch. When they are not, the higher is apt to appear the louder: for example, when a siren is being worked up it appears to get louder and louder as the pitch rises, though the expenditure of energy is no greater than before, but rather less. The mechanical intensity of a vibration, then, is not the measure of its loudness as judged by the ear; in fact the latter, within certain limits, appears to be inversely proportional to the wave-length. From a mathematical point of view, the intensity of a sound is inversely as the square of the distance, and directly as the square of the amplitude of vibration. Considering the former first, if any influence be radiated uniformly in all directions from a point, with no loss in transmission, the total intensity over the surface of a sphere, whose centre is that point, is the same whatever the radius, but over equal areas on different spheres it will vary inversely as the square of the radius. This follows at once from the expression for the surface of a sphere ($4\pi r^2$ where r is the radius), and is equally true of light, gravitation, etc. We observe that the amplitude must necessarily diminish inversely as the distance.

In practice, the intensity of ordinary sounds is affected to an extent not usually comprehended, by reflections from

walls or other hard surfaces, whether in a room or outside. The noise of traffic in a narrow street, for instance, or of games in a courtyard, is far more deafening than in a space not bounded by walls, other conditions being the same. So also in a large room, a speaker's voice is much more audible than it would be at an equal distance in the open air, but whether the syllables are recognisable or not depends on a number of other circumstances.

The amplitude or extent of excursion of the particles is also a factor in the intensity for obvious reasons. How excessively minute it may be and yet affect the ear is shown by a tuning-fork, which can be excited by the lightest touch, provided it is sudden enough: here the amplitude is amongst the smallest of measurable magnitudes. The intensity, however, is not proportional to the square of the amplitude of the *fork*, or its loudest sound would be more than ten thousand times its weakest, which cannot be admitted.

According to theory, the mechanical effect I of a pendular vibration of period t and amplitude a, in a medium of density ρ, is $\mathrm{I} = 2\pi^2\rho\frac{a^2}{t^2}$. When this is delivered with a velocity V, the quantity of energy per second is $2\pi^2\rho\,\frac{a^2}{t^2}\,\mathrm{V}$. From this formula Lord Rayleigh obtained a measure of the maximum amplitude of vibration in a shrill whistle as follows:—Air was supplied at the rate of 196 cubic centimetres per second under a pressure of 9·5 cm. of water; the frequency or number of vibrations per second was 2730, and the extreme range of audibility was 820 metres. Under the conditions of the experiment the velocity of sound (V) was 34,000 cm. per second, and the density of the air (ρ) was ·0013 (*i.e.* one cubic cm. of air weighed ·0013 gr.). The work done by the air passing through at the given rate $= 9{\cdot}5 \times 196 \times 981$ dynes, being the same as would be done by a mass of 9·5 grs. falling

by gravity through 196 cm. Equating the two expressions for work done, we have

$$2\pi^2 R^2 \rho \frac{a^2}{t^2} V = 9{\cdot}5 \times 196 \times 981,$$

whence

$$a = \sqrt{\left(\frac{9{\cdot}5 \times 196 \times 981}{2\pi^2 \times 82000^2 \times {\cdot}0013 \times 34000 \times 2730^2}\right)} = {\cdot}000,000,081 \text{ cm.}$$

It is assumed that all the energy is turned into sound, that there is no loss in transmission, and that the wave-fronts are hemispherical.

When the sounds are loud and the distances considerable, as in fog-signalling, many causes combine to interfere with the theoretical law of audibility at a distance, for instance the influence of wind, not only in helping to convey or retard the sound, but in altering the shape of the wave-fronts, so that with the wind the intensity is as great near the surface of the ground as it is anywhere, but against it, it is less than at some distance above.[1]

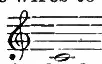
[1] Osborne Reynolds, *Proc. Roy. Soc.* 1874.

CHAPTER XIII

MUSICAL SCALE

Experiment XCVIII

Required.—Sonometer, etc.

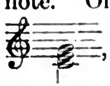
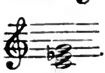
Tune both the wires to give any convenient note, say c' of the treble stave, , with 256 vibrations per second. Now, damp one wire by laying a feather on its middle point, and pluck or bow either of the halves. The note emitted is the higher octave, viz. c''. Sound the other wire at the same time, and if necessary make a slight adjustment to get the interval correct. The two sounds blend together in so perfect a manner as to show that one is merely a reproduction of the other on a higher scale. We have learnt that the number of vibrations of a wire (called n for short), other things being equal, is inversely proportional to the length, hence the octave of any note is performed by $2n$ vibrations. Next damp the wire at $\frac{1}{3}$ of the distance from either end, the note we get now from the shorter portion is g'', and is of course due to $3n$ vibrations per second. Continuing in this way we get the notes c''' e''' g''' c'''', etc., which form what is called a Harmonic Scale. Several of these may be recognised in the tone of the "open" wire (see Expt. CIII.).

EXPERIMENT XCIX

Now, using one wire only for the present, put the movable bridge at $\frac{2}{5}$ from one end, and pluck both parts together. An agreeable concord is produced, due of course to frequencies in the ratio of 3 : 2; this interval is called a Fifth in music for a reason we shall see presently. Notice that so long as the bridge is in the same place, the harmonious effect is preserved whether the tension be raised or lowered; or, in other words, it depends on the ratio between the frequencies, not on their absolute value.

Try in the same way the ratios 3 : 4, 4 : 5, 5 : 6, etc. Notice that as these approach unity, the more imperfect the harmony becomes.

EXPERIMENT C

Now from the two wires obtain the ratios 4 : 5 : 6 as follows. Both being in unison, put the bridge at the $\frac{2}{5}$ mark under one, and the $\frac{1}{2}$ mark under the other. Thus, supposing each wire is 100 cm. long, we get on one lengths of 40 and 60 cm., and on the other of 50 cm. Pluck all three together, and notice the purely harmonious effect. We have here what is called a Major Triad: again it depends only on the ratios, and not on the absolute values of the numbers, as we may see by tuning the wires to any other note. On a piano this corresponds to striking the notes , and in the following experiment the notes .

EXPERIMENT CI

Next obtain the ratios 10 : 12 : 15 or $\frac{1}{6} : \frac{1}{5} : \frac{1}{4}$ thus. The $\frac{2}{3}$ mark on one wire will give us 10 : 15, and to get

12 on the other, damp it at $\frac{12}{25}$ from one end, or what is the same thing, if the scale on the box is divided into 100 parts, move the bridge to 48 instead of 50, and pluck the shorter portion. We get now from the two wires together a Minor Triad; it at once strikes us as having a mournful character as compared with the other, but numerically the only difference between them is this, that whereas in the Major the interval 4 : 5 comes between the first two notes, and 5 : 6 between the second two, in the Minor it is just the reverse.

Upon these triads our Musical Scales are founded, thus: Call the notes, whose frequencies are as 4 : 5 : 6, C, E, G, respectively. Take the last of them as the foundation of a new triad, denoted by G, B, 2D, so that G : B : 2D are as $4 : 5 : 6 = 6 : 7\frac{1}{2} : 9$. We must not extend beyond the octave, and as the third member of this triad is represented by 9, whereas 8 is the limit, we call this note 2D instead of D. Lastly, using 2C as the *highest* note of a third triad, of which the other members are called F and A, we have F : A : 2C as 4 : 5 : 6. These are the tonic, dominant, and sub-dominant triads of the scale of C. To obtain the ratios between two successive notes C and D, for instance, we find that C : G = 2 : 3, and G : 2D as 2 : 3. Eliminating G, then C : D = 8 : 9. Proceeding similarly with the intervals C : E, C : F, etc., we get the complete scale or *gamut*, thus—

C	D	E	F	G	A	B	2C
1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

The *musical* intervals between these notes are denoted by their positions in the scale, thus C to D is a Second (usually called a tone), C to E a Third, C to F a Fourth, and so on.

Reducing all these fractions to a common denominator, and keeping the numerators only, they become—

$$\begin{matrix} 24, & 27, & 30, & 32, & 36, & 40, & 45, & 48, \\ C & D & E & F & G & A & B & C \end{matrix}$$

showing that the greatest difference is between A and B, and the least between E and F. Since the ratios and not the absolute differences are all-important in music we have, dividing each of these numbers by the preceding, and reducing—

$$\frac{D}{C}=\frac{9}{8},\ \frac{E}{D}=\frac{10}{9},\ \frac{F}{E}=\frac{16}{15},\ \frac{G}{F}=\frac{9}{8},\ \frac{A}{G}=\frac{10}{9},\ \frac{B}{A}=\frac{9}{8},\ \frac{C}{B}=\frac{16}{15}.$$

The interval $\frac{9}{8}$ occurs three times, while $\frac{10}{9}$ and $\frac{16}{15}$ each occur twice. The first is called a major tone, the second a minor tone, and the third a major semitone. The words major and minor have merely a relative signification. The interval from $\frac{9}{8}$ to $\frac{10}{9}$ or $\frac{81}{80}$ is called a *comma*, and for ordinary purposes may be neglected, so that on the whole there are practically only two different intervals, the tone and semitone.

So far, the scale is very incomplete; it contains only seven intervals, of which five are whole tones and two are semitones. Moreover, if any other note, say D, were taken as the foundation of a scale, and the series which bore to it the ratios $\frac{9}{8}$, $\frac{5}{4}$, $\frac{4}{3}$, etc., were calculated, we should find that most of them were different from the previous series. The minor triads also require consideration, and involve a different series again. Since in musical compositions the melody must at least be able to shift into any key, the number of notes required to preserve the theoretical relations throughout is very large, and quite beyond all practical convenience. To obviate these difficulties, instruments with fixed notes, like the piano, harp, and organ, are tuned in the system of equal temperament, where the octave is divided into twelve equal intervals, not one of which

is absolutely correct except the octave itself, but the mechanical advantages are infinite.

Instruments whose notes are adjusted by the performer, more particularly those of the violin tribe, as also the human voice, are not so restricted, and when guided by correct feeling, they can and do produce much more perfect effects than the piano or organ, a very minute shade making all the difference between complete precision and mere approximation. The Thirds suffer most, and the Fifths not quite so much, on the equally tempered scale.[1]

According to the convention of Helmholtz, the middle *c* of a piano with 256 vibrations per second is called c', the upper octaves are c'', c''', etc., and the lower ones *c* (128), C (64), $C_{,}$ (32), $C_{,,}$ (16). In the French convention the middle *c* is called ut_3, and so on.

Notes of the Scale

Calculating the successive lengths of wire required to give the notes of the scale, we find that calling C the note of the open wire, supposed 1 m. long, D is got from $\frac{8}{9}$ of it, and it must be damped at 88·9 cm.; for the other notes, E, F, G, A, B, C, the positions are 80, 75, 66·7, 60, 53·3, and 50 respectively.

Experiment CII

Lay a feather or the edge of a card on the wire at the points just found and observe the succession of notes. Keep the other wire in tune with the lowest note all the time, and sound it along with all these in turn. Notice the discords of the 2nd and 7th, and the harmony of the remaining combinations; also that a very slight difference in the position of the damper is sufficient to set up a perceptible degree of harshness.

[1] See Appendix II.

CHAPTER XIV

HARMONICS OR UPPER PARTIALS

WITH the exception of a tuning-fork and a wide stopped pipe, hardly any musical instrument gives a simple tone, *i.e.* one due to a pendular vibration; as a rule, several higher ones are present, due to some multiple of the rate of vibration of the fundamental. These tones are designated overtones, harmonics, or upper partial tones, while the fundamental is also called the prime tone or first partial. It is perhaps better to restrict the word harmonic to the partials whose frequencies are integral multiples of the fundamental; in practice they seldom are. In some cases the upper partials are very conspicuous, and overpower the fundamental, but the latter determines the pitch of the note. They are very prominent in reed pipes, sirens, bells, and instruments of percussion, but by no means absent from the tones of a piano, organ, violin, etc.

In the less musical of these instruments many of the partials are inharmonic with one another, and the effect is to this extent discordant, but in strings they form, in theory, a harmonic series, the frequencies following the natural numbers. We can name the intervals from the fractions $1, \frac{9}{8}, \frac{5}{4}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \frac{15}{8}, 2$; for taking the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12, we see that the first *upper* partial is the octave of the fundamental, the second bears to this the ratio 3 : 2, and is therefore the Fifth above the

octave; the third is to the second as 4 : 3, interval of a Fourth, the fourth to the third as 5 : 4, a major Third, and so on. The only prime numbers concerned in the gamut are 2, 3, and 5, so that the sixth upper partial, with a frequency seven times that of the fundamental, has no place in it. This is the first note which introduces a slight discord; the eighth partial is still more objectionable, and should be got rid of. In a piano this is done by letting the hammers strike at about $\frac{1}{7}$ to $\frac{1}{9}$ from one end of the wires, and so creating an antinode where these partials require a node. Under these circumstances they are not evoked (see Expt. CIV.). Proceeding a little farther, we come to the intervals 10 : 9 and 11 : 10, these are also inharmonious combinations, but by this time the intensity of the partials is generally very small.

Several partials may be recognised in a piano-wire thus. Raise the damper of the note *c* in the bass clef , strike the octave somewhat forcibly, and then release it: the note is now continued from the lower wire. The observation may easily be repeated with the twelfth and double octave. Now strike the note *c* alone, and listen for the upper partials one after another.

Upper Partials of a String or Wire

Experiment CIII

Required.—Sonometer and bow.

Tune both wires accurately to the same note, press a feather lightly on the middle of one, and draw the bow along either half. Now stop this wire altogether, and the octave or first upper partial will be given out from the other. Excite either of the wires, and the octave can be heard by directing the attention to it; just as when several people are talking at once, it is possible to listen to one alone by an effort of will. Repeat the experiment, laying

the feather on one of the points of trisection, and bowing the shorter portion; the passive wire will continue the sound after the other is stopped. Here again this note can be detected when the open wire alone is vibrated.

Experiment CIV

It is exceedingly instructive here to verify an observation of Dr. Thomas Young, viz. that when a wire is bowed or plucked at any point, all the upper partials which require that point for a node vanish.

Having given the ear the suggestion of the note to be listened for (say the second upper partial), draw the bow or pluck the wire at a point of trisection, and it will be at once clear that this note is no longer present. One can tell with certainty in fact whether a wire has been excited at this particular point or not.

Continue the observations as far as the 5th or 6th upper partial. Also notice the effect of laying a feather at different points when the whole wire is vibrating.

Experiment CV

Required.—Sonometer; short length of stiff wire; light hammer faced with felt, or a piece of cork fastened to the end of a penholder.

Pluck the wire aside with a hard metal point, notice the jangling noise it gives, especially when plucked near one end. This is due to high upper partials which, in this mode of excitation, have great intensity, even overpowering the fundamental. Hence we see that it is not a matter of indifference where the motion begins. Strike it in the middle with a soft hammer, the tone is poor and hollow, being wanting in certain upper partials, viz. the first, third, fifth, etc., because these require a node in the middle, and the best mode is that long ago arrived at by pianoforte

makers, viz. to use a soft hammer, and to strike at $\frac{1}{7}$ or $\frac{1}{9}$ from the end.[1]

[1] Helmholtz (*Sensations of Tone,* chapter v.) has calculated the intensities of the partial tones of a string according to the manner of excitation. The following is abridged from his table :—

Number of the Partial Tone.	Excited by Plucking.	Struck by Hard Hammer.
1	100	100
2	81·2	324·7
3	56·1	504·9
4	31·6	504·9
5	13	324·7
6	2·8	100
7	0	0

CHAPTER XV

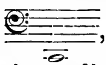
UPPER PARTIAL TONES OF WIND INSTRUMENTS

WE have seen (p. 81) that the possible modes of vibration of an open pipe are in the order 1, 2, 3, 4, etc., and of a closed pipe 1, 3, 5, etc. Hence it might be expected that the partial tones would also follow these ratios. Unless the pipes are narrow and cylindrical, however, the notes to which they resound are not the harmonic upper partials of the prime tone; that is to say, if we tested the resonance of a pipe for tuning-forks of different pitches, those which it reinforced would not in general have frequencies which were exact multiples of the frequency of the fundamental. Hence wide pipes have few upper partials. When overblown the note jumps up an octave, or a twelfth or double octave. In organs the quality of tone is brightened on several of the stops by sounding the upper partials on other and smaller pipes blown simultaneously with the principal one.

In reed pipes the reed itself moves with a simple harmonic motion, and if held in front of a resonator tuned to the same note, gives a sound like a tuning-fork. But when it closes and opens an aperture the motion of the air is highly complex, and the pipe reinforces such components as coincide with its own upper partials. They can be separately recognised until the intervals between them become very close, *i.e.* up to about the sixteenth partial, according to the pitch of the prime.

In the clarinet, oboe, and bassoon, technically called "wood-wind," there is a thin wooden reed, single in the former, but double in the two latter. The clarinet has a cylindrical tube with a "bell" at the far end: it gives the partials of an open pipe, and the effective length of the column can be varied by opening and closing holes in the side by means of levers or the tips of the fingers.

The oboe has a conical tube, the reed is double, and is formed of the outer layer of a siliceous grass. The two blades are placed at an angle with each other: they are very much smaller than the clarinet reed, and are attached to the end of a narrow tube, which is removed for safety's sake when the instrument is not in use. The natural harmonics are the octave, twelfth, double octave, etc., of the prime tone, but the missing notes are supplied as in the clarinet.

The bassoon is similar in many respects to the preceding, but the tube is doubled on itself so as to give an 8-foot tone, the C, , of the staff notation.

The flute is a cylindrical wooden pipe, in which a sheet of air from the lips is directed against the edge of a wide hole in the wall near one end. Various smaller holes are bored at different points along the tube, these can be opened separately or together as desired. The instrument has a compass of three octaves, of which the highest is produced by overblowing.

In the bugle and post-horn (as in brass wind instruments in general) the performer's lips form the reeds, and are pressed against a small cup-shaped metal mouthpiece. The tube is quite open from end to end, and the four notes it yields form a common chord, and are all upper partials of the fundamental. The other brass wind instruments, especially the horns, depend very largely upon upper partials for their usefulness; in all of them the lips act as reeds, and the change in the rate of vibra-

tion is brought about by altering their tension and the force of the air. By opening and closing valves and changing the "crooks" it is possible to obtain a complete scale and play in several different keys; the tone is extremely penetrating owing to the intensity of the higher partials, and a single brass instrument can immediately be distinguished among any amount of "strings" and "wood-wind."

In the trombone there are no valves, but their place is taken by a "slide," a U-shaped metal tube which can be moved backwards and forwards along two parallel tubes of rather smaller diameter, which act as guides to it. The position of the slide is judged by the performer, who learns by practice exactly where to leave it to produce any given note. The other brass instruments, saxophone, euphonium, bombardon, etc., do not call for any remark here.

Experiment CVI

Upper Partials of Organ Pipes

Required.—Organ-pipe; resonators.

Using a set of resonators, analyse the sound of an open and a closed organ pipe, giving the note of the lowest of them. The pipe can be tuned as on p. 80. With a closed pipe, which, as we have seen, can only contain $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$, etc., of a wave, the first upper partial is the Fifth above the octave, while the octave itself and the double octave are wanting: in an open pipe there are no omissions.

Sounds that have a brilliant and penetrating character, like those of a reed pipe or siren, are very rich in upper partials, those from which they are almost entirely absent, *e.g.* a tuning-fork or large stopped organ pipe, are sweet and agreeable, but dull in character.

When the fundamental is itself very high, the upper partials soon get beyond the limit of hearing, hence it is

difficult or impossible to recognise the instrument in such cases, apart from incidental noises. Thus an extremely high note on a violin can hardly be distinguished at a distance from the same note on a pipe, and so on.

The analysis of the tones of other wind instruments can be made in a similar way.

Upper Partials of the Human Voice

The tones of the human voice differ from those of other instruments in the same way as these differ among themselves, *i.e.* in the number and force of the upper partials of the prime tone.

The mechanism consists of several parts: the chest performing the function of a bellows, the vocal cords, which are really semicircular elastic *membranes*, acting as reeds, and the cavities of the throat and mouth as resonators. By an effort of will the disposition of these parts can be so arranged that any vowel sound can be sung to any note within the compass of the voice. Each vowel sound consists of the fundamental or prime tone of the note, with upper partials which vary in number and strength for the different sounds. The analysis can be made by a trained ear to some extent, but with resonators it is more complete and certain. A piano can be used with considerable success as follows. Press down the forte pedal, thereby raising all the dampers. Sing one vowel sound after another into the top of the instrument and listen to the echo: each is reproduced quite clearly, though of course not with human expression. This proves that there is nothing in a vowel sound which is in any way different from the ordinary tones of instruments. Even a whistle is returned faithfully. We cannot, however, tell yet which wires are vibrating, but by the following method some notion of the correctness of the theory may be obtained. Suppose we are dealing with an upright piano, the bar

which holds the dampers must be removed, and also the frame carrying the hammers, etc., so as to expose all the wires. Tilt the instrument by letting the upper part rest on a chair, so that the wires are all inclined at a convenient angle to the ground. With a horizontal grand, of course, some of this labour can be spared. Prepare a number of riders of paper or thin mica and put them on the wires which may be expected to vibrate, and now on singing to a particular note they begin to slide down the moment the agitation begins, while those on other wires remain stationary. The amplitude of the shorter wires is so extremely small, however, that the experiment is inconclusive as regards them.

In any case it does nothing more than introduce the subject of the constitution of vowel sounds, which is an extremely difficult one. The harmonic theory, viz. that the partials are harmonics of the fundamental, varying in pitch with it, and that the vowel lies in the predominance of certain of these, is considered unsatisfactory. That each vowel has partials of fixed pitch which confer the particular quality upon it is also doubtful. There is a third view, that the predominant partials in each case are those which are nearest in pitch to the resonating cavities of the mouth.

Graphical Representation of Compound Tones

The general principle is the same as that described on p. 98, and consists in taking the algebraical sum of the ordinates of the curves at different points along the axis. Instead of taking two curves at random, we now take one to represent the prime tone, a second of half the wave-length and say half the amplitude, corresponding to the first upper partial, another of one-third the wave-length and amplitude, and so on, all in the same phase at starting. In Fig. 68 a combination of six such harmonic curves is

shown, but it must be understood that in air the amplitudes in no way approach the dimensions represented,

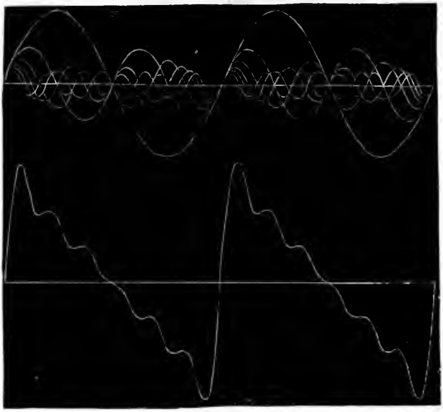


FIG. 68.

being always an exceedingly small fraction of the wavelength. They usually diminish as the partials get higher, but this is not necessarily the case.

Analysis of Compound Tones

The converse problem, to find the simple harmonic components of any regular periodic curve, is beyond the power of graphical methods, but it can be done, with certain limitations, by a machine called a Harmonic Integrator. That such an analysis is possible is shown by an exceedingly important theorem due to Fourier, which proves that any such curve can always be resolved, in only one way, into simple harmonic curves whose period is $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., that of the given curve. The connection of this theorem with Acoustics is made through a principle enun-

ciated some thirty years later by G. S. Ohm, that only a pendular vibration excites in the ear the sensation of a simple tone (*i.e.* one free from upper partials). It appears, further, that every compound tone is resolved by the ear into its simple harmonic constituents by a mechanism residing in the cochlea. This organ contains many thousands of fibres, called after Corti, their discoverer, which are arranged in a kind of spiral arcade, and it was at one time supposed that each of these was tuned to and vibrated in sympathy with a particular note whenever the occasion presented itself.[1] The difficulties, mechanical and other, which beset this explanation are insuperable, but some such analysis is accomplished, or no effort of attention would enable us to pick out the partials of a compound tone as objective realities. The vexed question of phase here assumes great prominence. We have seen that two or more curves can be compounded in a great number of ways, according as they start together, or at slightly different points. Now since the graphic differences have an analogy in the pressures at different parts of a sound-

[1] This idea, generally attributed to Helmholtz, can be traced much farther back. In a book entitled *Philosophy*, in dialogue form, published in 1763 by Benjamin Martin, occurs the following passage:—

"*Euphrosyne.* I observe it [the cochlea] with a great deal of pleasure, and as this winding passage grows narrower towards the summit, I apprehend the fibres of the auditory nerve, displayed through the same, may be supposed to have some resemblance to the system of strings in a harpsichord, and that in this part we may expect to find the true seat or cause of concords and discords, or of the harmony and dissonance of sounds."

"*Cleonicus.* Your conjectures perfectly coincide with those of the most experienced naturalists, who have always conceived this to be the case . . . for in such an infinite variety in the lengths of nervous cords it will always happen that some or other of them will be in unison, or some other concord, with the vibrations of the air impressed, and others in discord with the vibrations of the said air."

The Marquis Corti published his researches on the organ of hearing in mammals in 1851. Previously to this, theories as to the nature of audition were merely conjectures, and even now the point is unsettled, though there is a much firmer basis to work on. (See p. 210.)

wave, it would appear that the sensations excited in the ear must vary accordingly. Helmholtz held that they did not, and brought forward several arguments in support of this opinion. In order to combine a number of simple tones he contrived an instrument consisting of twelve tuning-forks with resonators, giving the prime and upper partial tones of a note of 120 vibrations per second. All these were caused to vibrate simultaneously by suitable electro-magnets, but were not audible unless their resonators were opened. A small disc, attached to a lever, which was worked from a keyboard, enabled this to be done for each at will, while the resonator itself could be moved up to or away from the fork. By slightly shading the aperture, a difference of phase was brought about, so that all possible combinations could be tried. No variation in the character of the resultant tone could be detected. The question is of vital importance to the theory of audition, because if the analysis by the ear takes place on the principles above indicated, phase differences should be without influence. On the other hand, several observations, such as the "revolving" character of slow beats, seem to point to the conclusion that they do have an effect. Dr. Koenig (see Appendix III.) has made many more experiments tending in the same direction, and at present the matter hangs in suspense.

CHAPTER XVI

INTERFERENCE AND BEATS: COMBINATIONAL TONES

THE phenomena of interference and beats spring from the same cause, viz. the meeting of wave-systems; in the former case the effect is continuous, in the latter intermittent. Several instances of each have been alluded to, *e.g.* an ear held over a Chladni's plate hears little or no sound at certain points (p. 55), neither does a tuning-fork excite any sensation when held in a particular position near the ear. Beats are noticed whenever two sonometer wires are brought into unison, and in other similar cases.

EXPERIMENT CVII

Interference produced by Resonators

Required.—Fork; two resonators placed at right angles.

The fork is struck and held in the angle between the jars, and then rotated on its axis. When the prongs are in a certain position, the sound is quenched because the waves meet in opposite phases, a condensation from one jar synchronising with a rarefaction from the other, so that the resultant effect is nil. If either of the jars be covered up, the fork is heard plainly. The interference can also be brought about by using a single jar, and turning the fork to the proper position.

EXPERIMENT CVIII

Interference of Two Organ Pipes

Required.—Two 4-feet stopped organ pipes of the same make.

Mount them on the same wind-chest near to one another and blow the bellows: instead of the sound being twice as loud as before it becomes very much feebler, because the pulsations adjust themselves so that one pipe is in a state of condensation while the other is in the opposite condition. The effect is liable to be interfered with by trifling circumstances. In organ-building this mutual influence of pipes is familiar, and where necessary, means are taken to prevent it.

EXPERIMENT CIX

Interference (continued)

Required.—Helmholtz siren (see p. 49).

Turn the handle, which works the upper disc, through 45°, so that in the common circle of twelve holes the upper set are 15° removed from the lower. Now when the instrument is blown, the puffs below coincide with stoppages above, and in consequence the prime tone is much enfeebled. As the sound is very complex, however, nothing approaching to silence results, but certain of the upper partials become very prominent, viz. those whose period is not interfered with by the change.

EXPERIMENT CX

Interference produced by Surface Waves

Required. — Shallow glass dish containing mercury; funnel, filter-paper.

Fold a filter-paper in the usual manner, and cut off a small piece from the point. Having put it in the funnel,

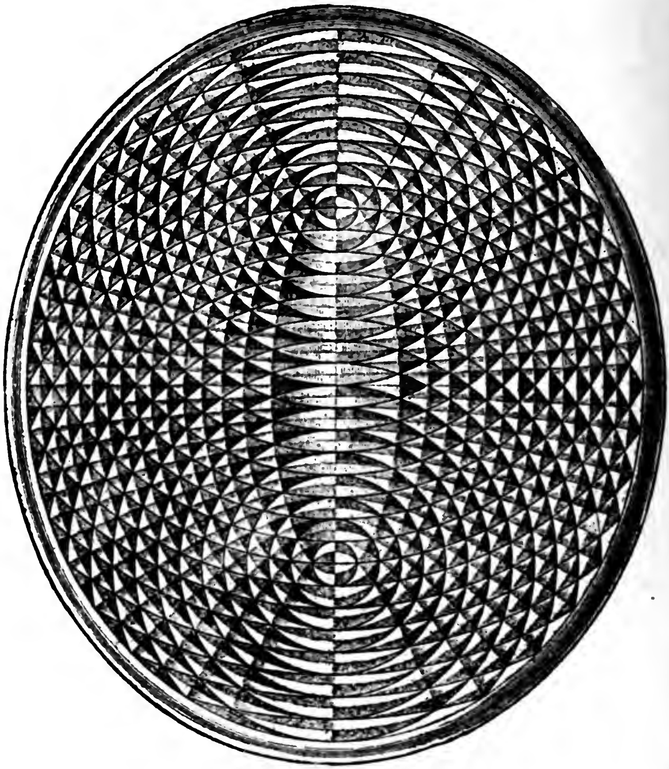


Fig. 69.

pour mercury in till it issues in a fine stream, and contrive matters so as to avoid splashing. Unless the agitation takes place exactly in the centre of a circular dish, a

complex pattern is produced on the surface, due to the interference of direct and reflected waves.

In Fig. 69 the effect in an elliptical trough is shown, the stream falling at one of the foci. Notice the 15 confocal hyperbolas, these mark the places where the original level is unchanged. Also notice that reflections from the sides throw the centres of the circles from one focus to the other, so that the figure is perfectly easy to reproduce by simply drawing a number of circles round two points chosen at random, and diminishing the radii by successive steps (= $\frac{1}{2}$ a wave-length).

Experiment CXI

Interference of Direct and Reflected Sound-Waves

Required.—A Galton's whistle.

Make it give a sound well within the range of audibility, and hold it some 8 or 10 cm. from a wall. Move the ear backwards and forwards on a level with the whistle, and it will be found inaudible at some points, and louder than usual at others. This is due to a state of things analogous to that in the previous experiment. It must be noticed that the places of silence are antinodes, where the density is constant. If reflection were perfect, a number of hyperboloidal surfaces could be traced round the whistle as a focus, where no sound could be heard.

Experiment CXII

Interference produced by Difference of Path

Required.—Apparatus shown in Fig. 70.

It consists of an arrangement of brass tubes such that a series of sound-waves produced by a fork at A is led in two directions and subsequently reunited. On one side

the path can be varied because the U-tube slides backwards and forwards as in a trombone. When the difference amounts to half a wave-length, no sound is audible at C, the two series destroying one another entirely. If the paths could be measured accurately, this would be a convenient way of finding the velocity of propagation (wave-length × frequency), but obviously it does not lend itself to this purpose. A difference of $1\frac{1}{2}$, $2\frac{1}{2}$, etc., wave-lengths would be equally effective if the slide were long enough. The apparatus may be made in glass by connecting two T-pieces to the corresponding U-tubes by indiarubber joints. Suppose it is made for a fork of frequency 512, the half wave-length corresponding to this is about 33 cm. at ordinary temperatures, and the tubes must be cut accordingly.

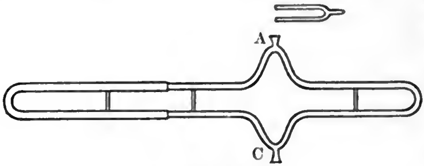


FIG. 70.

EXPERIMENT CXIII

Hopkins's Forked Tube

The instrument is shown in Fig. 71. The main tube is capped by a box over which a membrane is stretched, while below it divides into two branches. These are put over alternate or adjacent sections on a Chladni's plate, with the result that sand sprinkled on the top is agitated in the former case, but not in the latter. This is, of course, due to the waves meeting in the same or in opposite phases.

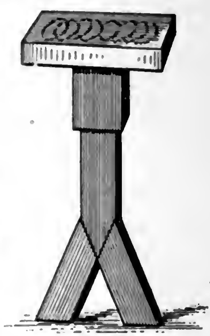


FIG. 71.

Beats

When two notes very nearly in unison are sounded together, the ear is not able to hear either of them separately, as it is when they are at an interval apart, but it is conscious of a throbbing effect, an alternation of sound and partial or complete silence to which the name of Beats is given. Strictly speaking, beats are the sounds, not the alternations.

They are of great assistance in tuning instruments, because they cease altogether when unison is reached, and it is possible to recognise them when they take many seconds to complete. The method recommended by Scheibler, however, is to make both the sounds which it is desired to bring in tune to beat 4 times per second with one of fixed pitch. Here, by continuing the observation long enough, the error may be indefinitely reduced, whereas in the ordinary method this is not possible.

Beats of this class are termed beats of imperfect unison, or more shortly, unison beats. There are also beats of imperfect consonance, frequently called Smith's or consonance beats, and thirdly, beats of overtones or upper partials. The subject involves many very difficult questions, and there are several points still unsettled in regard to them.

Graphical Representation of Beats

Draw a series of 11 waves (Fig. 72), and another of 10, within the same limits of space. Find the shape of the compound curve by adding or subtracting the ordinates as usual; it will appear as in the third line. The amplitude is at times considerable, but presently sinks to zero, and so on. In the case of sound-waves a similar composition occurs, and supposing the amplitudes to be very small in comparison with the wave-lengths, they would, in the compound curve, alternate between twice that of the components (supposed equal) and nothing. The

intensity being proportional to the square of the amplitude, varies between 4 and 0, hence it is that beats can be heard when the sounds producing them are exceedingly faint.

If the condition of things represented in Fig. 72 were repeated 20 times in a second, *i.e.* if the frequencies were 200 and 220, there would be 20 beats per second, and, in general, the number is $n - n'$, where n and n' are the frequencies, but there are some reservations. For example, notes of 40 and 80 vibrations per second would not beat at all, because one is the octave of the other, and

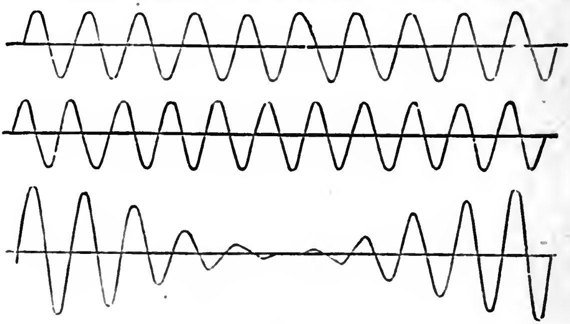


FIG. 72.

so on. Hence the mere diagram of resultant displacements does not represent the whole matter. According to Bosanquet[1] the interval at which two notes can be heard separately, in addition to the beats, is from 2 to 3 commas, *i.e.* $\left(\frac{81}{80}\right)^2$ or $\left(\frac{81}{80}\right)^3$, in the middle of the scale. With a smaller difference than this, a single note is heard, whose pitch is intermediate between those of the primaries, and which varies in intensity from 0 to a maximum. When the difference is greater the intermediate note is not heard, but the ear effects a separation, as explained on p. 160.

[1] *Phil. Mag.* June 1881, "On the Beats of Mistuned Consonances of the Form $h:1$."

Experiment CXIV

Number of Beats from Two given Notes

Required.—Helmholtz siren.

Obtain the unison sound from the 12 holes in the upper and lower disc, and maintain it at a steady pitch. Turn the handle so as to make the upper box meet or follow the rotating disc; then every time it has gained or lost $\frac{1}{24}$th of a circumference as compared with the fixed disc in the lower box, the puffs from above and below issue in opposite phases, and destroy one another. Hence the number of beats per second is the same as the number of gains or losses of this fraction of the circumference. But this is also the number of puffs added or subtracted per second. Hence the number of beats is equal to the difference between the frequencies. Whatever the rate of rotation, the same rule holds.

No other instrument proves this principle so easily, for on the sonometer, for instance, though it is very easy to produce beats, we should have to limit them to 5 or 6 per second to make sure of the counting, and the errors in determining the frequencies might amount together to nearly as much. With two tuning-forks, whose frequencies are accurately known, the observation is of course made perfectly easy.

Experiment CXV

Beats producing a Visible Effect

Required.—Sonometer; glass tube drawn out fine and connected to a water-tap.

Clamp the tube in a retort stand, and direct it so that the water shall fall into a trough. Turn the tap so as to obtain a feeble jet; its dimensions may vary within wide limits, but a suitable size is soon obtained by trial. Put

the wires nearly in unison with one another, and sound them both; the jet will be seen to undergo pulsations corresponding to the beats. When two large forks which beat are screwed on to the same board the throbbing is perceptible to the touch.

Beating Distance of Two Notes

When two notes originally in unison are separated by a gradually-increasing interval, the beats are at first slow, then grow rapid and pass through a period of maximum jarring, and ultimately disappear when the nearest concord is reached. The difference between the frequencies at this moment is called the "beating distance." It might be thought that since a minor Third is the first consonance, the beating would always cease at this point, but it is not so; it varies at different parts of the scale. Professor Mayer has compiled a table giving the beating distance for notes of several pitches. It is useful in many ways, *e.g.* to tell which of the upper partials of a chord will beat together.

MAYER'S TABLE OF BEATING DISTANCES

Note.	Frequency.	Beating Distance.	Duration of Sensation.	Interval.
C	64	16	$\frac{1}{16}$ sec.	$\frac{80}{64}$ = major 3rd
c	128	26	$\frac{1}{26}$,,	$\frac{154}{128}$ = minor 3rd
c′	256	47	$\frac{1}{47}$,,	$\frac{303}{256}$ = minor 3rd less $\frac{1}{4}$ semitone
g′	384	60	$\frac{1}{60}$,,	$\frac{444}{384}$ = tone + $\frac{1}{32}$
c″	512	78	$\frac{1}{78}$,,	$\frac{590}{512}$ = minor 3rd less $\frac{1}{2}$ semitone
e′	640	90	$\frac{1}{90}$,,	$\frac{730}{640}=\frac{9}{8}+\frac{1}{64}$ } practically a tone
g″	768	109	$\frac{1}{109}$,,	$\frac{877}{768}=\frac{9}{8}+\frac{1}{59}$ } practically a tone
c‴	1024	135	$\frac{1}{135}$,,	$\frac{1159}{1024}=\frac{9}{8}+\frac{1}{35}$ } practically a tone

In column 4 the fractions are the reciprocals of the numbers in column 3. If, for instance, 26 beats per second fuse into a continuous sensation, it shows that at this part of the scale the duration of an impression is $\frac{1}{26}$th of a second. At higher pitches it is less, and at lower ones greater. So on a piano, a shake executed

rapidly low down in the bass cannot be followed, whereas nothing is easier in the upper or middle parts.

Experiment CXVI

Verification of the above

Required.—Sonometer.

Tune one wire to $c' = 256$, and starting with the other nearly in unison, tighten it till the beats of the prime tones are no longer discernible. Now move the slide along the first wire till it is exactly in tune with the other: take its length with a beam compass, then calculate its frequency from the law of lengths. For example, suppose it was 100 cm. long at first when tuned to 256 v.s., and 84·2 cm. when tuned to the other wire: then this wire executes $\frac{84 \cdot 2}{100} \times 256 = 304$ v.s., or 48 more than before. As the difference of vibrations is considerable, a small error in determining the length does not affect the result to the same extent as if there were only 5 or 6 beats to be observed.

Beats of Imperfect Consonance (Smith's Beats)

Beats can not only be heard from two notes nearly in unison, but also when they nearly make a consonance. For example, if we sound notes of 100 and 151 v.s. together, making a slightly imperfect fifth, what will be the result? Supposing the two wave systems to start in the same phase, then the 49th of the first and 74th of the second will differ in time by $\frac{74}{151} - \frac{49}{100} = \frac{1}{15100}$ of a second, that is, they will be practically in agreement once more, and at some intermediate point before and after this moment they must be in opposite phases. Hence there will be 2 beats per second. If the rates were 100 and 152 there would be coincidences 4 times in

a second, and hence the same number of beats. The general rule is that if $\frac{a}{b}$ be the ratio of the consonance, expressed in its lowest terms, and if the lower note corresponding to a be sharpened to the extent of one vibration a second, the number of beats is b, but if the other note be sharpened to the same extent, then a beats are heard. For let the prime tones have 500 and 300 vibrations per second, so that the interval is the major 6th (5 : 3), then if we make the 500 into 501 we get an approximate coincidence from 5×300 and 3×501, where the difference is 3. But if we make the 300 into 301, the difference is now to be sought between 5×301 and 3×500, where it is 5 instead of 3.[1] With any interval it will be seen that the number of beats does not depend on the absolute number of vibrations, but on the numbers composing the fraction, and whether the mistuning amounts to 1, 2 or more vibrations per second.

On a Helmholtz siren it is possible to arrange the following intervals, so that either the upper or lower notes can be taken on the upper disc, and hence be sharpened or flattened by one or more vibrations per second. Minor 3rd, $\left.\begin{matrix}12\\10\end{matrix}\right\}$ or $\left.\begin{matrix}15\\18\end{matrix}\right\}$; Fifth, $\left.\begin{matrix}15\\10\end{matrix}\right\}$ or $\left.\begin{matrix}12\\18\end{matrix}\right\}$; Octave, $\left.\begin{matrix}16\\8\end{matrix}\right\}$ or $\left.\begin{matrix}9\\18\end{matrix}\right\}$. One revolution of the handle turns the upper box through one-third of a revolution or 120°; to add or subtract one puff per second it must be turned at different rates according to which row of holes is open.

On the innermost row (9 holes) the angular distance between two adjacent holes is 40°, hence the box must move one way or the other through 40° in one second, and the handle through 120° in this space of time.

According to Bosanquet,[2] the beats of mistuned consonances are always on the lower of the two notes

[1] This mode of reasoning leads to the same result as the other.
[2] *Phil. Mag.* June 1881.

concerned, and the limit to which they can be followed diminishes as the pitch rises. For some of Koenig's experiments on this subject see Appendix III.

Experiment CXVII

Beats of Upper Partials

Required.—Tuning fork; sonometer.

Suppose the fork gives $c''=512$: make the wire give $c'=256$, and listen for its first upper partial, which will be the octave. Sound the fork gently: if the tuning be perfectly accurate no beats will be heard, but on throwing the wire ever so little out of tune, a slow rise and fall can be detected owing to a difference of a few vibrations per second between the fork and wire.

To pursue this subject far requires a special training of the ear. We may point out that amongst chords in general the sources of dissonance are numerous. Take for instance the minor triad 120, 144, and 180, and their upper partials whose frequencies are less than 1000 per second, and we have the following list :—

Primes and Upper Partials.			Differences.
120	...	...	...
...	144	...	24*
...	...	180	36
240	...	...	60
...	288	...	48*
360	...	360	72
...	432	...	72
480	...	...	48
...	...	540	60*
...	576	...	36*
600	...	...	24*
720	720	720	120
840	...	...	120
...	864	...	24*
...	...	900	36*
960	...	...	60*

In column 4 are the differences or number of beats per second, those marked with a star being within the beating distance according to Mayer's table (p. 170). There are thus no less than eight sets of beating notes, hence on instruments whose upper partials are powerful, and especially on those tuned in equal temperament, the discord of such a triad is considerable. Moreover, we have not considered the combinational tones, which would introduce fresh sources of dissonance.

Experiment CXVIII

Combinational Tones

Required.—Organ pipes giving some musical interval other than the octave. A pair of singing flames, or an accordion, or even a sonometer, may also be used.

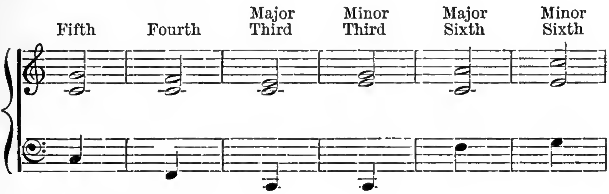
Suppose we take the interval of a Fifth. Sound it with some intensity, and by careful attention a third note will be heard, an octave below the lower of the two primaries; these should be about the middle of the scale.

Some assistance in picking out the note is afforded by sounding it gently on a fork or wire, so as to give a suggestion of what is required.

Tones of this class are called Differential tones, and their frequency is always the difference between the frequencies of the primaries. They arise, according to Helmholtz, as a necessary consequence, on theoretical grounds, from the amplitudes not being negligible in comparison with the wave-lengths. The graphical representation then no longer explains the whole effect, and not only differential tones are produced, but also *summational* ones, due to a frequency equal to that of the primes added together. Considering the former alone for a moment, two notes separated by a Fifth (3 : 2) will give a differential tone of 1; when they are a Fourth (4 : 3) apart, the

difference is again 1, but it is now two octaves below the higher of the primes.

Continuing the observation for other consonant intervals, the scheme is as follows, where the primes are represented by minims, and the differential tones by crotchets.



The reality of these tones is indisputable, but seeing that the frequency is equal to that of the beats, a ready explanation offers itself, that they are beats fused into one another, or beat-tones. Against this view it has been urged by Tyndall that beats are heard when the primaries are very faint, differential tones only when they are loud: also by Helmholtz, that they are a necessity on the mathematical theory.

It is also possible to hear both beats and differential tones together in certain cases.[1]

The summation tones are, of course, always higher than the primaries. An interval of a Fifth (3 : 2) will have a summation tone of 5: this stands to the higher of the two primes in the relation of 5 : 3, and is hence a Sixth above it. They cannot be heard when tuning-forks are used, but do appear with a polyphonic siren.

The following is the scheme for consonant intervals less than an octave: the primes are shown in minims as before,

[1] Dr. Koenig maintains that *both* kinds of combinational tones are merely beat-tones of the primes or of upper partials, and as a mere matter of arithmetic they would always be accounted for on this hypothesis. See, however, a paper by Rücker and Edser "On the Objective Reality of Combinational Tones," *Phil. Mag.* April 1895.

and the summation tones by crotchets. In the last two cases the true note lies between the two figured ones.

Dr. Koenig has made a large number of observations on this point, but the question at the present time cannot be said to be settled (see Appendix III.).

CHAPTER XVII

SOUNDS MAINTAINED BY HEAT AND SENSITIVE FLAMES

EXPERIMENT CXIX

Singing Flames

Required.—Hydrogen apparatus; several tubes about 2 cm. diameter, and of different lengths (Fig. 73).

The bottle contains zinc and water, and a little strong sulphuric acid is admitted through the funnel. The aperture at which the gas is lighted must be narrow and have thick walls, otherwise they melt very soon and almost fall together. Before applying a light it is very necessary to collect a test-tube full of the gas to see that it burns quietly, otherwise the apparatus will be blown to pieces. Having obtained a small flame, bring one of the wide tubes down over it, and on reaching a certain position, a loud and clear musical note will be emitted, which can be maintained for any length of time. The experiment also succeeds with a flame of coal gas, but as the vigour of combustion is less, it is more easily extinguished, especially in a narrow tube. It has the advantage, however, of being more luminous.

With tubes which are not too long in relation to their diameter, the note given out is nearly that of an organ pipe of the same length (not quite, because the air inside is hot), but when a certain limit is exceeded it is impossible to

obtain the fundamental, the column always subdividing so that one of the upper partials is emitted.

The altered appearance of the flame should be noticed: it seems stiffer, and is in a state of vibration. This is

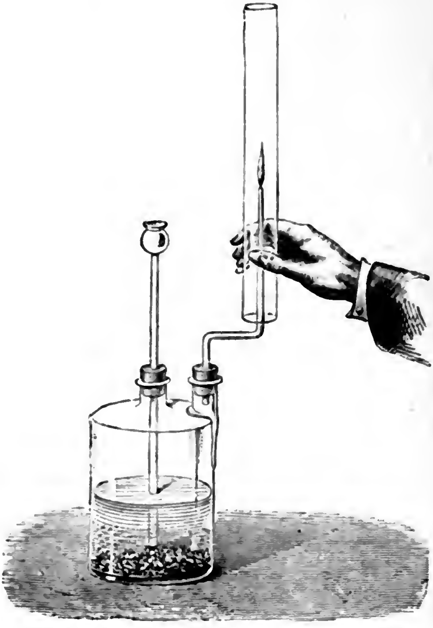


FIG. 73.

immediately evident on using a coal-gas flame, turned up till it just begins to be luminous, and viewing it in a plane mirror turned backwards and forwards through a small angle by a motion of the hand. With a hydrogen flame the room has to be darkened before it can be seen.

When the flame is too low down in the tube, it will not begin to sing of itself, but if the same note be sounded in its vicinity, it does do so, at first feebly and then more loudly.

By putting a cardboard slide on the top of the tube, so as to prolong it more or less, a certain amount of tuning may be effected, and such a source of sound is sometimes far more convenient than a fork, which requires exciting at intervals, or an organ pipe, which requires continuous blowing (see p. 47). A pair of flames, adjusted to some musical interval, are of great use in connection with beats and combinational tones for a similar reason.

It was discovered by Kastner[1] that two flames inside the same tube would sing so long as they were separate, but on bringing them together they ceased to do so. He devised an instrument called a "pyrophone," which was founded on this principle, and in its latest form had a ring burner in each tube, which, when turned low, gave a number of separate flames; but the sound could be stopped by turning them up so that they met as in an ordinary argand burner. The invention is, however, merely a scientific toy.

The theory of a singing flame is by no means so simple as it would appear. When a little cotton wool is pushed nearly to the top into the tube supplying the gas, a flame can be obtained as easily as before, but it refuses to sing, because stationary waves in this tube are a necessary part of the phenomenon, and they are now prevented from forming.

Again, the enlargements and contractions of the flame must synchronise with the pulses of air, so that heat is supplied and withdrawn at the same rate as the condensations and rarefactions follow one another. The flame is, therefore, more powerfully affected the nearer it is to the centre of the tube, because there is a node at or near this

[1] *Comptes Rendus*, 1873.

point. The phenomena of singing flames were first critically studied by Sondhauss.

Experiment CXX

Singing Flames (continued)

Required.—Hydrogen apparatus; gas-jar or phosphorus globe.

Having obtained a small flame, bring the jar or globe over it: a low and deep note is heard, not however under all circumstances, as some adjustment of the size and position of the flame may be necessary. As before, it depends on the flame enlarging and contracting, in agreement or otherwise with the changes of temperature induced by the vibration. "When the transfer of heat takes place at the moment of greatest condensation, or of greatest rarefaction, the pitch is not affected. If the air be at its normal density at the moment when the transfer of heat takes place, the vibration is neither encouraged nor discouraged, but the pitch is altered. Thus the pitch is *raised* if heat be communicated to the air a quarter period before the phase of greatest condensation, and *lowered* if it be communicated a quarter period after the phase of greatest condensation."[1]

Experiment CXXI

Singing Flames (continued)

Required.—Glass tube drawn out slightly and connected with a gas supply pipe; wire gauze; large metal tube of almost any size, say 60 cm. by 4 cm.

Hold the gauze at such a distance over the jet of gas,

[1] Rayleigh, *Theory of Sound*, vol. ii. p. 226.

that when lit on the upper side, the flame is blue and flickering. Lower the wide tube on to it, a harsh and screaming sound will be produced, which may in some cases attain a great intensity.[1] A glass tube may be used instead of the metal one, but is apt to crack. A Bunsen burner with a rose on the top also gives the effect; it is usually extinguished after a short time by the violence of the vibration.

Here, as in similar cases, the maintenance of the sound is due to a transfer of heat from the flame to the air; the pitch is affected or not according as this transfer takes place at the moment of greatest condensation (or rarefaction) or at some other moment.[2]

Before bringing the tube down on the gauze it may be observed that the flame is sensitive, and responds to noises made by coughing, knocking the table, etc.

Experiment CXXII

Rijke's Tube

Required.—Hard glass tube about 40 cm. long and 3 cm. in diameter; gauze.

Cut a piece of wire gauze of such a size that it will stay in the tube without falling. Push it up some little way, and make it red hot by a flame of gas or of spirit on cotton wool. Remove the flame, and almost immediately a loud and clear sound is heard, whose pitch depends chiefly on the length of the tube. The general principles involved are the same as those already indicated. The "incandescent" gas lights sometimes emit a note when the flame is turned partly down.

[1] The observation can be traced back to 1842. See *Nature*, vol. x. p. 286.

[2] Rayleigh, *loc. cit.*

EXPERIMENT CXXIII

Sound of a Cooling Bulb

Required.—Quill glass tubing; blowpipe flame.

Select a piece about 14 cm. long, and blow a bulb 2 cm. in diameter upon one end of it. During the cooling a sound is sometimes heard; whether it will happen or not in any particular case cannot be predicted, but when it does it can be renewed by heating the bulb again.

Lord Rayleigh[1] gives as the explanation the flow of air from parts where it is cooler to where it is hotter, heat being received at the phase of greatest condensation and given up at the phase of greatest rarefaction. "The adjustment of temperature takes time, and thus the temperature of the air deviates from that of the neighbouring parts of the tube, inclining towards the temperature of that part of the tube from which the air has just come. . . . In order that the whole effect of heat may be on the side of encouragement, it is necessary that, previous to condensation, the air should pass not merely towards a hotter part of the tube, but towards a part which is hotter than the air will be when it arrives there. On this account a great range of temperature is necessary for the maintenance of vibration, and even with a great range the influence of the transfer of heat is necessarily unfavourable at the closed end, where the motion is very small. This is probably the reason of the advantage of a bulb."

EXPERIMENT CXXIV

The Trevelyan Rocker

Required.—Instrument shown in Fig. 74.

It consists of a brass or copper bar with a groove run-

[1] *Theory of Sound*, vol. ii. p. 231.

ning along its lower surface, and having a long handle attached, at the end of which is a ball of wood. A ring or triangular prism of lead is also requisite.

Heat the bar in a flame, making it very hot, as judged by the hand, but not nearly red hot, place it as shown in the figure, and it will give a clear ringing note,

FIG. 74.

which under favourable conditions is maintained for a considerable time.

This curious effect is due to the bar being alternately tilted from one side to the other, by small humps raised by expansion during the momentary contact.

The phenomenon can be produced with other metals, bar and block being both the same or different. Of a very large number of substances examined by Tyndall,[1] rock-salt, though not metallic, appeared to be the best, vibrations being kept up till the (metal) rocker was below blood-heat.

EXPERIMENT CXXV

Sensitive Flames

Required.—Glass tube about 1 cm. in diameter, and contracted to $1\frac{1}{2}$ mm. at the orifice, which should be slightly **V**-shaped.

Connect the tube with a large gas supply pipe so as to obtain as long a flame as possible. It will be found to shrink considerably when certain noises are made in its vicinity, such as knocks, clinks, hisses, etc. Using a gas-bag giving a higher pressure than the mains, it is possible

[1] *Phil. Mag.* July 1854.

to obtain a flame over 50 cm. long, which will respond to almost every syllable uttered by the voice. When a gas cylinder is used the flow is apt to be interrupted by eddying currents, or by hissing sounds made by the gas itself, unless a long connecting tube is employed.

The best result is obtained by using a pin-hole burner made of steatite, as described by Tyndall;[1] the flame then attains a length of 60 m. and is extremely sensitive. The seat of sensitiveness is at the orifice, as may be shown by concentrating sounds on different parts of it through a funnel. It must in all cases be near the point of flaring, so that a slight change in pressure, such as is produced by a sound-wave, will momentarily bring about this condition.

A fish-tail flame in a concert room or church is sometimes observed to send out tongues when certain notes are sounded, in fact the whole series of phenomena had their origin in an observation of this kind by Professor Leconte in 1858. The flame is not really necessary, as smoke jets are even more sensitive; at the same time they are very troublesome owing to the difficulty of avoiding draughts.

Experiment CXXVI

Sensitive Flames (continued)

Take a large Bunsen burner and turn the gas about half down. Make a loud noise near it, say by tapping a mortar; the flame will be extinguished. Here a sudden wave is generated in the tube, acting as a resonator, and interferes with the current of gas to the point of extinction. A special burner illustrating this effect was introduced some years ago by Messrs. Fletcher of Warrington: the tube was horizontal, and a non-luminous flame issued from an annular space at one end. It was supplied with additional air from the middle of this space (which was

[1] *Sound*, chap. vi.

open below) and could be easily extinguished by clapping the hands, etc., when the pressure was properly adjusted.

EXPERIMENT CXXVII

Sensitive Flames (continued)

Required.—Tube fitted up as in Fig. 75.

The wide tube is about 12 cm. × 2 cm., at the lower end it is stopped by a cork perforated to admit a gas supply tube, and a conical flame appears at the top. To prevent a tube cracking under such circumstances it is convenient to prolong it by a thin sheet of mica rolled up and placed inside.

I. "Lower the inner tube till the flame is on the point of roaring. It will now be found very sensitive to noise. Snapping the fingers at a distance of 8 or 10 yards will cause it to contract fully $\frac{4}{5}$ of its height. The most suitable flame for this is about 6 inches high.

II. "Adjust the gas to give a flame about $4\frac{1}{2}$ inches high, and gradually raise the inner tube. A point will be reached at which the flame becomes sensitive, not to noise, but note, and it will be found to respond to a certain note by dividing into two portions, and while this note is produced it will continue divided. . . .

III. "Arrange two singing flames to give the proper note; the flame divides. Now make one a little sharper than the other so as to beat slowly. The wings alternately recede and coalesce.

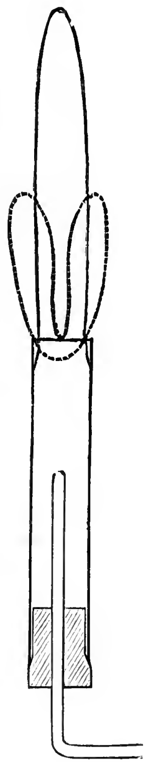


FIG. 75.

IV. "Using the whistle, blow so hard as to produce

higher octaves of the responding note. The flame will be unaffected, as though in perfect silence."[1]

Manometric Flames

These have assumed great importance, chiefly as an adjunct to lecture experiments, but also on their own merits as detectors of sound. They were introduced by Dr. Koenig.[2] The apparatus is very simple (Fig. 76),

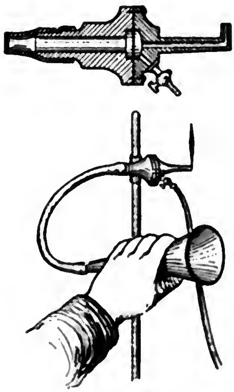


FIG. 76.

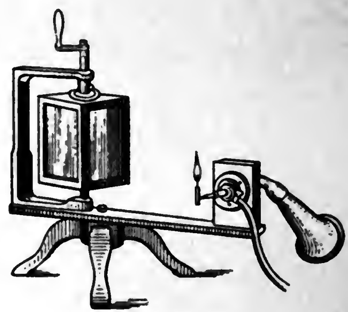


FIG. 77.

consisting of a small cavity divided into two parts by a flexible membrane of oiled silk or goldbeater's skin. A current of gas enters the capsule by a supply pipe, and burns at another outlet. The membrane is exposed on the opposite side to aerial vibrations from the voice or other source of sound, and the flame oscillates up and down in response. It is viewed by reflection from four plane mirrors attached to the sides of a wooden cube which is

[1] R. H. Ridout, *Nature*, vol. xv. p. 119. See also Barrett, *Nature*, 3rd May 1877 (vol. xvii.).

[2] *Phil. Mag.* Jan. and Feb. 1873.

rotated on a vertical axis (Fig. 77). The appearances presented under the influence of different instruments and

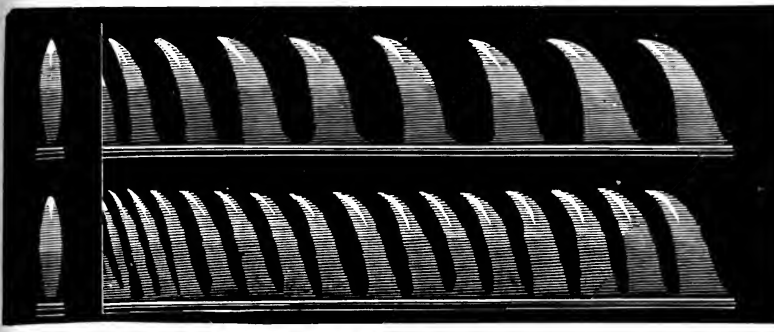


Fig. 78.

vowel sounds, etc., are extremely interesting and suggestive, every different quality of sound producing its own figures.

These capsules may be conveniently connected with

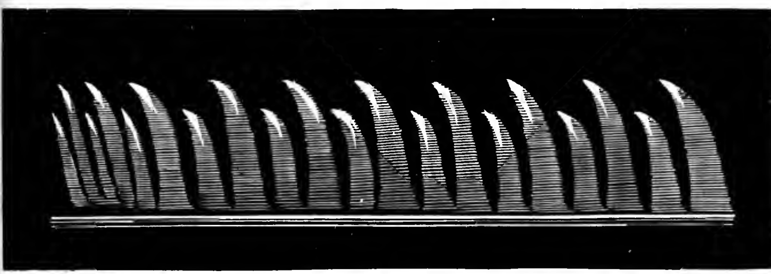


Fig. 79.

resonators, or let into the walls of organ pipes. In the latter case they are affected at the nodes, like the ear itself, because the impulses only act on one side.

A working form of the apparatus may be made from a

large cork which has been cut in two at right angles to the axis. Each part has a conical cavity cut in it, they are then glued with their bases together, and separated by a diaphragm made of goldbeater's skin, or thin indiarubber from a toy balloon. One cavity communicates with the source of sound by a hole through the apex; the other has two tubes leading into it as already described (Weinhold's *Experimental Physics*).

In Fig. 78 are seen the appearances of the flame, first when a simple tone is sounding, and secondly when the octave alone is sounding. Fig. 79 shows the effect when both the tones are operating.

CHAPTER XVIII

WATER-JETS AND MISCELLANEOUS OBSERVATIONS

Experiment CXXVIII

Required.—Glass tube drawn out to a moderately fine point, and connected with a water-tap; sonometer.

The tube must be contracted slowly and not violently, or the smoothness of the jet is interfered with by eddies.

On obtaining a gentle stream of water, which is most convenient when directed slightly upwards, it is found to be affected by almost any note from the sonometer wire or the voice. The best result will probably not be obtained at once, as a certain relation between the sound and the size of jet is necessary. To the eye it appears as if the jet had been stiffened, and instead of the drops spreading and falling irregularly they now keep in the same path, and widen and contract at regular intervals, giving an appearance like twisted glass. The sound of the falling drops is also different. It may happen that a small branch is thrown off from the main one, or that two separate streams form instead of one. Lastly, as has been already mentioned (p. 169), the jet throbs under the influence of beats.

When a stream of water flows out of a sponge, kept saturated from a tap above, it does not respond to sounds, but when it issues from a glass tube drawn out as described, it is affected.

In the former case it splits up after a certain interval into a succession of alternate large and small drops; this is a result of surface tension, just as when a soap bubble is pulled apart by placing two waxed and soaped wire rings in contact with it, and separating them, a minute bubble always floats away. The large drops oscillate visibly, having their longest axis now vertical and now horizontal. When the stream issues from a drawn-out tube, the aperture not being perfectly circular, it oscillates slightly, even before dividing; under the influence of aerial vibrations it begins to split up into drops sooner than before, and the oscillations are always in the same phase at the same part of the curve, or, in other words, the wavy surface is stationary with respect to the curve. Under proper conditions, two separate and slightly divergent streams are formed, and it may happen that some of the smaller drops, by always colliding with larger ones at the same spot, are knocked away laterally; when this is the case they fall much more nearly in a vertical direction, owing to their feeble inertia.

When the drops are directed on a stretched membrane, such as a tambourine, they cause it to give out the same note as the one whose influence they are under.

If a metal cylinder containing some water be put under a tap, and the water turned on a very little, it is noticeable that the character of the sound varies with successive drops, according to the different phases in which they strike the surface.

The following curious production of sound has been known for a long time:[1] A glass tube 2 m. long by 6-8 cm. diameter is closed at one end by a metal plate, in the centre of which is a hole whose diameter is equal to the thickness of the plate. On being filled with water,

[1] *Phil. Mag.* July 1854, a posthumous paper by F. Savart "On some Acoustic Phenomena produced by the Motion of Liquids through short efflux Tubes."

the efflux takes place in a periodic manner, and a tone is heard, feeble and confused at first, but acquiring force as the pressure diminishes till it reaches a certain limit. Beyond this the intensity decreases, and in some cases disappears altogether. But as the level sinks, the tone regains force and passes through another maximum of intensity, and so on. Certain precautions are necessary to obtain the effect, *e.g.* the plate must be accurately horizontal and the edges of the hole sharp.

Jets of liquid flowing into another liquid are also acted on, but by much graver sounds, having, say, 20 to 50 v.s. In the *Theory of Sound*, § 370, so often quoted from, some experiments of this kind are described, water tinged with potassium permanganate being allowed to pour into pure water. It takes a sinuous curve, or divides into two parts connected by a sheet, etc.

Experiments on Audition

I

Press a finger into the orifice of each ear, a murmuring sound, called the "susurrus" of the muscles, can be heard. According to the experiments of Dr. Haughton it is due to vibrations at the rate of 32 per second. It is louder when the muscles are in a state of contraction.

II

Press the stem of a sounding tuning-fork on any part of the head, and stop up one ear. The sound now appears far louder in that ear than in the other one, a definite localisation being produced, which is absent when both ears are opened. When both ears are closed, sounds made by the voice appear very much louder than before. Under the same circumstances the sound of a fork placed on the head appears louder in the ear to which it is nearer.

III

Find the personal limit of audibility by a Galton's whistle. The gravest note for which the instrument is graduated has a length of half an inch, and being a stopped pipe it corresponds to a frequency of about $1120 \times 12 \times 2 = 26880$ (the velocity in feet per second at 15° C. being 1120). The plug can be screwed forward till it reaches the lip, but a pipe does not speak well unless its length is greater than its diameter. The feebleness of the sound also makes it uncertain whether or not the absolute limit has been reached.

IV

The outer tube of the ear, the *meatus auditorius externus*, has a pitch of its own, which may be discovered by shading the ear with the palm, and striking notes on a piano till the right one is reached. It is usually in the highest octave.

V

Cover one ear with the curved palm, press a vibrating tuning-fork against the back of the same hand, and observe how surprisingly loud it appears, no doubt because of the confined space into which the sound is led.

VI

Take two forks of the same pitch, place them on opposite sides of the head, and while holding one in a fixed position, rotate the other so that it is alternately audible and inaudible. The fixed one is only heard at the moments when the other is not heard.

The following observations are worthy of mention :—

In order to determine the direction of a sound, two ears are of great assistance. If the listener be allowed to turn

his head round, however, he can judge tolerably well by using only a single ear. He can (with both ears) tell immediately whether the sound of a tuning-fork comes from the right hand or the left, but not whether it is in front or behind. Curiously enough, there is no difficulty about doing this with the voice. If small, flat sheets of cardboard be placed round the ears at different angles, the sense of direction is misled, because the natural degrees of audibility on the two sides are interfered with.

A hoarse or screaming sound, such as that of a siren, cannot be shut out by stopping the ears.

Sounds nearly in unison beat with one another when they are led into separate ears, though there are no moments of absolute silence.

A grave sound has the power of extinguishing a high one, but the converse is not true. Mayer[1] employs an organ pipe and a tuning-fork of higher pitch, and finds that the latter ceases to be heard even when it is sounding quite loudly; but on the other hand, when a fork lower in pitch than the pipe is used, it can be heard just as long with or without the continuous sound. It may be urged, however, that in musical compositions the melody is, as a rule, pitched higher than the accompaniment, and that in an orchestra no amount of "strings" will extinguish the sound of a voice, or a flute, or a brass wind instrument.

The production of sound by allowing an intermittent beam of light (or rather of heat) to fall on a metal diaphragm was discovered in 1880 by Mr. Graham Bell,[2] who was then investigating the change in the electrical resistance of selenium under the action of light. Tyndall substituted absorbent vapours, such as sulphuric ether, chloride of methyl, aqueous vapour, etc. These were contained in

[1] "Acoustical Researches," *Phil. Mag.* 1876.

[2] The following references in *Nature* may be consulted :—Tyndall, 17th February 1881 ; Preece, 24th March 1881 ; Tyndall, 5th January 1882 ; and Mercadier, *Phil. Mag.* January 1881.

flasks, from which a tube led to the ear, and the beam from a lantern was concentrated on them from a concave mirror. The intermittence was produced by a revolving perforated disc, every access or withdrawal of heat setting up a vibration.

Diffraction of Sound

Using as the source of sound a bird call,[1] and a sensitive flame as detector, Lord Rayleigh[2] has succeeded in obtaining effects resembling the lights and shadows produced by light waves when they pass round the edge of a small circular disc at right angles to the direction in which they are travelling. Thus rings of sound and silence were detected when a circular plate, 38 cm. in diameter, was placed 70 cm. from the source, which gave waves of $1\frac{1}{2}$ cm. and was 25 cm. from the flame. Further, "when a suitable circular grating cut out of a sheet of zinc is interposed between the source of sound and the flame, the effect is many times greater than when the screen is removed altogether."

"Polarisation" of Sound

The phenomena of plane polarisation in Optics are, as is well known, due to the different aspects of the planes of vibration of the ethereal waves towards reflecting or refracting surfaces. In the case of longitudinal vibrations it would appear impossible that anything of the kind could take place, but the following observations of Wheatstone[3]

[1] Two metal plates, about 2 cm. in diameter, bored centrically with a small hole, are fixed at a small distance from one another (this may be as little as 1 mm. when the plates are of thin brass). Air is blown through from a short supply tube, and a note of very high pitch is elicited, even reaching 50,000 vibrations per second.

[2] *Phil. Mag.* vol. ix. p. 281, 1880; *Theory of Sound*, vol. ii. p. 143; *Nature*, 28th June 1888.

[3] Reprint of Scientific Papers, No I., written in 1823.

show that analogous effects can be obtained. He took a straight rod, one end of which was connected with a sounding-board, and placed a tuning-fork at the other end; on bending the rod to a right angle, the plane of oscillation being perpendicular to the plane of the angle, the sound could scarcely be heard; when the two branches were parallel, it was nearly as loud as before. By multiplying the number of right angles it could be completely stopped. On turning the fork round, the rod not being bent, it was heard most plainly when the vibrations took place parallel to the rod, and least plainly when at right angles.

"The phenomena of polarisation may be observed in many corded instruments; the cords of the harp are attached at one extremity to a conductor which has the same direction as the sounding-board; if any cord be altered from its quiescent position so that its axis of oscillation shall be parallel with the bridge or conductor, its tone will be full; but if the oscillations be excited so that their axis shall be at right angles with the conductor, its tone will be feeble. By tuning two adjacent strings of the harp unisons with each other, the differences of force will be sensible to the eye in the oscillation of the reciprocating string according to the direction in which the other is excited."

A singular observation, which is easily verified, is described as follows. "When a tuning-fork, placed perpendicularly to a rod, communicating at one or both extremities with sounding-boards, and caused to oscillate with its vibrating axis [or plane of vibration] parallel with the rod, moves along the rod, preserving at the same time its perpendicularity and its parallelism, the vibrations will not be transmitted while the movement continues, but the transmission will take place immediately after it has remained motionless. When the tuning-fork moves on the upper edge of a plane perpendicular to a sounding-board,

the vibrations rectilineally transmitted will not be influenced by the progressive motion."

RECENT ACOUSTICAL INSTRUMENTS

The invention of the telephone has been followed by that of a large number of instruments all directly dependent on it. In the Photophone of Mr. Graham Bell there is a selenium cell, consisting of a number of thin circular brass discs separated by sheets of mica of somewhat smaller diameter, the spaces between being filled up with selenium, so that on the circumference brass and selenium follow one another alternately. The 1st, 3rd, 5th, etc., discs of brass are in metallic connection, as also the 2nd, 4th, 6th, etc.; hence the current passes from one set to the other through the intervening layers of selenium. The transmitter is a thin, silvered disc of glass, supported in a mouthpiece. It vibrates under the influence of sound-waves, and thereby undergoes alterations in curvature. A powerful beam of light is sent on it from a lantern, thence it passes to a parabolic mirror placed at a considerable distance, and is condensed on the selenium cell which is held in the focus. A galvanic cell furnishes the current, which also includes a telephone. An increase in the illumination reduces the resistance of the selenium cell, and *vice versa;* these changes, though excessively minute under the circumstances, furnish the alterations in current strength necessary to affect the telephone disc.

In the Water-jet Amplifier of Dr. Chichester Bell[1] a vertical column of acidulated water falls on the end of an upright rod containing a platinum wire in its axis. A telephone and galvanic cell are required as before. The form of the jet, whose length must be carefully adjusted, is altered by vibrations reaching it through the supports or

[1] "On the Sympathetic Vibrations of Jets," *Phil. Trans.* 1886.

otherwise, and these alterations increase or diminish the current of electricity, thereby agitating the telephone disc, which, of course, is placed close to the ear.

In the Microphone, invented by Professor Hughes, the alterations in current strength are produced by the variations in conductivity of one or more carbon rods, which are shaken by the sounds to be conveyed. This instrument is so well known in the various forms of commercial loud-speaking telephones as not to need description here, but it may be remarked that the "microphone contacts," as they are called, take place even in a tube of iron filings in a varying electrical field, and that the theory that the variations in conductivity are produced by simple pressure is wrong, or at least does not express the whole truth. According to Professor S. P. Thompson they are to be ascribed to exchanges of molecular energy between the particles in contact, or separated only by minute distances.

The Phonograph, invented by Edison in 1877, is now very well known. In its later forms it consists of a cylinder of hardened wax, made to revolve at a constant rate of several turns per second by a well-governed electromotor. This cylinder is minutely indented along a helical line by a sapphire or chalcedony point which is attached to the centre of a thin glass diaphragm, set in a suitable mouthpiece. Sounds received upon the diaphragm impress a to-and-fro movement on the cutter, causing it to scrape away the wax to a depth varying with the intensity. When a reproduction is desired, another sapphire point, joined by a curved piece of steel wire to a second glass diaphragm, is brought to the starting-point, and the cylinder is rotated at the same rate as before. So perfectly is the original motion reproduced, that not only can a person's voice be recognised, but the sound of instruments accompanying a chorus can be plainly detected. Dr. M'Kendrick[1] and his assistants have lately devised a

[1] *Proc. Roy. Soc. Ed.* vol. xxi. p. 201 (1896).

means of amplifying the curves on the wax by an arrangement of levers carrying a kind of siphon recorder: they are thus shown magnified nearly 1000 times in height and about 35 times in length. The record of a word consists of "a long series of waves, the number of which depends (1) on the pitch of the vowel constituents in the word, and (2) on the duration of the whole word or of its syllables individually. . . . The number of waves producing a word is sometimes enormous. . . . A record of the words 'Royal Society of Edinburgh,' spoken with the slowness of ordinary speech, showed over 3000 vibrations."

When a vowel sound is sung into the mouthpiece, and a different rate of rotation is imposed on the cylinder during the reproduction, it might be expected that the truth or falsity of the fixed pitch theory would be at once settled; but hitherto observers have not been unanimous as to whether the vowel is altered or not.[1]

[1] *Theory of Sound*, ii. p. 474.

APPENDIX I

"A STRING which is pulled aside by a sharp point, or the finger nail, assumes the form (Fig. 80) A before it is released. It then passes through the series of forms B, C, D, E, F, till it reaches G, which is the inversion of A, and then returns through the same to A again. Hence it alternates between the forms A and G. All these forms, it is clear, are composed of straight lines, and on expressing the velocity of the individual points of the strings by vibrational curves these would have the same form. Now, the string scarcely imparts any perceptible portion of its own motion directly to the air. Scarcely any audible tone results when both ends of a string are fastened to immovable supports, as metal bridges, which are again fastened to the walls of a room. The sound of the string reaches the air through that one of its extremities which rests upon a bridge standing on an elastic sounding-board. Hence the sound of the string essentially depends on the motion of this extremity through the pressure which it exerts on the sounding-board. The magnitude of this pressure, as it alters periodically with the time, is shown in Fig. 81, H, where the height of the line *hh* corresponds to the amount of pressure exerted on the bridge by that extremity of the string when the string is at rest. Along H*h* suppose lengths to be set off corresponding to consecutive intervals of time, the vertical heights of the broken line above or below H*h* represent the corresponding augmentations or diminutions of pressure at those times. The pressure of the string on the sounding-board consequently alternates, as the figure shows, between a higher and a lower value. For some time the greater pressure remains unaltered, then the lower suddenly ensues, and

likewise remains for a time unaltered. It is the alternation

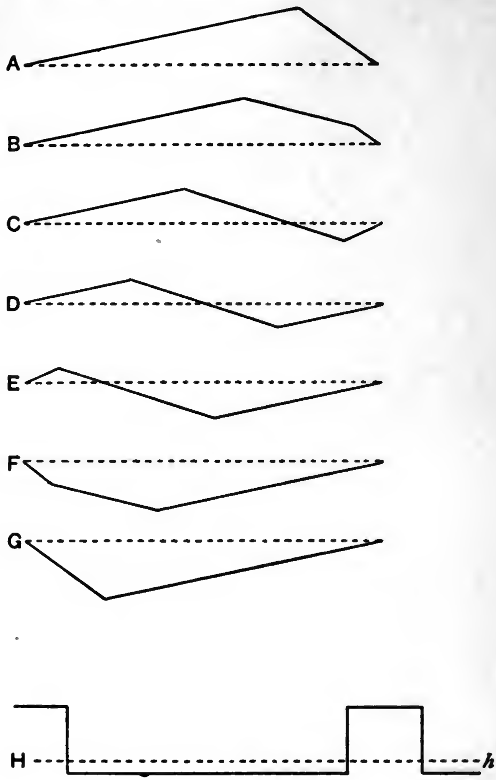


Fig. 80.

between a greater and a smaller pressure which produces the sound in the air" (Helmholtz, *Sensations of Tone*, p. 86).

APPENDIX II

RELATIONS OF NATURAL AND TEMPERED INTERVALS

On the equal temperament or chromatic scale, with 12 notes to the octave, each interval is $\sqrt[12]{2} = 1{\cdot}059$ . . . the amount of error in several instances being shown as follows :—

Theoretical Scale.	Equal Temperament Scale.
Minor semitone $\frac{25}{24} = 1{\cdot}042$	Chromatic semitone $= 1{\cdot}059$.
Major semitone $\frac{16}{15} = 1{\cdot}067$	
Minor third $\frac{6}{5} = 1{\cdot}200$	Three semitones or $(1{\cdot}059)^3 = 1{\cdot}189$.
Major third $\frac{5}{4} = 1{\cdot}250$	Four semitones $(1{\cdot}059)^4 = 1{\cdot}260$.
Fourth $\frac{4}{3} = 1{\cdot}333$	Five semitones $(1{\cdot}059)^5 = 1{\cdot}335$.
Fifth $\frac{3}{2} = 1{\cdot}500$	Seven semitones $(1{\cdot}059)^7 = 1{\cdot}498$.

The tempered Fourth is thus a little too sharp and the Fifth a little too flat; but these errors are small compared with those of the major and minor Thirds.

One of the chief advantages of having twelve equal intervals to the octave is that the Fifth is more nearly accurate than it would be with any other number, unless it were a very much greater one.

The simplest way of dealing with intervals is to take as unit the 100th part of a tone, or the 600th of an octave, and to use logarithms, thereby substituting addition and subtraction for multiplication and division. The log of the 600th root of 2 is $\frac{.30103}{600}$, or $\frac{1{\cdot}003433}{2000}$, which is easier to work with. To express any interval, such as the Fifth, divide the log of the ratio by this number, or what comes to nearly the same thing, multiply

it by 2000. If greater accuracy is desired, diminish the result by $\frac{1}{300}$, because $1{\cdot}003433 = 1\frac{1}{300}$ nearly.

For example, the Fifth becomes $2000 \times (\log 3 - \log 2) = 352$, or making the correction, 351. Again, the comma becomes $2000 \times (\log 81 - \log 80) = 10{\cdot}8$, which we may call 11, and so on. The complete enharmonic scale is then represented as follows:—

Scale.	Interval.		Log × 2000.	Equal Temperament.
C				
	Comma	$\frac{81}{80}$	11	...
C♯	Minor semitone	$\frac{25}{24}$	35	50
D♭	Major semitone	$\frac{9}{8} \times \frac{24}{25}$	67	
D	Second	$\frac{9}{8}$	102	100
D♯	Augmented 2nd	$\frac{9}{8} \times \frac{25}{24}$	137	150
E♭	Minor 3rd	$\frac{6}{5}$	158	
E	Major 3rd	$\frac{5}{4}$	193	200
F♭	Diminished 4th	$\frac{4}{3} \times \frac{24}{25}$	214	
E♯	Augmented 3rd	$\frac{5}{4} \times \frac{25}{24}$	228	250
F	Fourth	$\frac{4}{3}$	249	
F♯	Augmented 4th	$\frac{4}{3} \times \frac{25}{24}$	284	300
G♭	Diminished 5th	$\frac{3}{2} \times \frac{24}{25}$	316	
G	Fifth	$\frac{3}{2}$	351	350
G♯	Augmented 5th	$\frac{3}{2} \times \frac{25}{24}$	386	400
A♭	Minor 6th	$\frac{8}{5}$	407	
A	Major 6th	$\frac{5}{3}$	442	450
A♯	Augmented 6th	$\frac{5}{3} \times \frac{25}{24}$	478	500
B♭	Minor 7th	$\frac{15}{8} \times \frac{24}{25}$	509	
B	Major 7th	$\frac{15}{8}$	544	550
C♭	Diminished 8ve	$2 \times \frac{24}{25}$	565	
B♯	Augmented 7th	$\frac{15}{8} \times \frac{25}{24}$	579	600
C	Octave	2	600	

To use the table, suppose we wish to find the interval between D♯ and G: the numbers opposite these notes are 137 and 351, which by subtraction give 214; from another part of the table we learn that this corresponds to a diminished Fourth. On the equal temperament scale it would appear as a major Third.

Reckoning the Fifth and Fourth (and octave) as perfect consonances, they become diminished or augmented when multiplied by $\frac{24}{25}$ or $\frac{25}{24}$, but the terms major and minor are not applied. Among the others, or imperfect consonances, a major

interval cannot be diminished, nor a minor one augmented. The augmented perfect intervals are sometimes called pluperfect, and an augmented or pluperfect Fourth (C to F♯) is also known as a Tritone. Observe that, contrary to the usual rule, F♭ is graver than E♯, and C♭ than B♯.

APPENDIX III

DR. KOENIG'S RESEARCHES

Dr. Koenig has arrived at conclusions differing from those of Helmholtz as to the mode of production of beats of different intervals, and the existence of beat tones, *i.e.* tones produced by the coalescence of beats. His experiments do not admit of being repeated by the student, but may be summarised as follows.

When two forks differing by a wide interval are sounded together, two sets of beats are audible, equal in number to the positive and negative remainders obtained by dividing one of the frequencies into the other. For example, forks of 40 and 74 vibrations per second give one set of 34 and another of 6 beats per second. The former of these is the ordinary remainder when 40 is made to go into 74, the other is what has to be subtracted to give 74 when it is made to go twice ($2 \times 40 - 74 = 6$). On Helmholtz's theory the latter would be accounted for by the upper partial of 40 beating with 74, but this explanation is not always satisfactory.

Again, from forks of 64 and $106\frac{2}{3}$ v.s. one set of beats at the rate of $42\frac{2}{3}$ per second is audible, and another at the rate of $21\frac{1}{3}$. The former of these may in this and similar cases be called the *inferior*, and the latter the *superior* set.

With frequencies of 64 and 112 the inferior set (48) is not heard, but the superior (16) comes out plainly: similarly, 64 and 120 give 8 strong beats.

The following table gives the results with forks of higher pitches, the beats now blending into a continuous tone :—

Frequencies.	Ratio.	Inferior Beat-Tone.	Superior Beat-Tone.
2048, 2304	8 : 9	256	...
2048, 3840	8 : 15	...	$Ut_3 = 256$
2048, 3072	2 : 3	$Ut_5 = 1024$	$Ut_5 = 1024$
2048, 2816	8 : 11	$Sol_4 = 768$	$Mi_5 = 1280$
2048, 3328	8 : 13	$Ut_3 = 256$	$Ut_3 = 256$
1024, 2304	4 : 9	$Ut_3 = 256$	...

No explanation of these can be given on the supposition that they are all differential tones.

Moreover, when the forks Ut_6 and Sol_6 (2048 and 3072) were taken, and the latter loaded with wax so as to give 3070 vibrations, beats were heard at the rate of 4 per second, because the remainders now become 1022 and 1026. This proves the reality of the beat-tones, for otherwise they would not themselves produce beats.

Proceeding to other intervals, the following table shows the frequencies of the forks employed, the ratios of these numbers in their lowest terms, the number of beats (positive remainders), and the names of the corresponding notes :—

Prime Tones.	Ratio.	Beats.	Note.
4096 and 3840	16 : 15	256	c_3
,, 3968	32 : 31	128	c_2
,, 4032	64 : 63	64	c_1
,, 4048	256 : 253	48	g_{-1}
,, 4056	512 : 507	40	e_{-1}
,, 4064	128 : 127	32	c_{-1}
,, 4070	158 : 157	26	...

All these resultant tones are audible except the last, which is just below the limit of hearing, and produces a fluttering sensation.

With regard to beat-tones, then, they have the exact pitch due to the frequency of the beats themselves, they interfere and give secondary beats, and the same number of beats always

gives the same beat-tone irrespective of the interval between the primaries.

Dr. Koenig has also made bars of steel of rectangular section, which when struck in one direction give one note, and at right angles to that direction another note. A particular one gave 2048 and 2304 for the two directions: on striking it obliquely

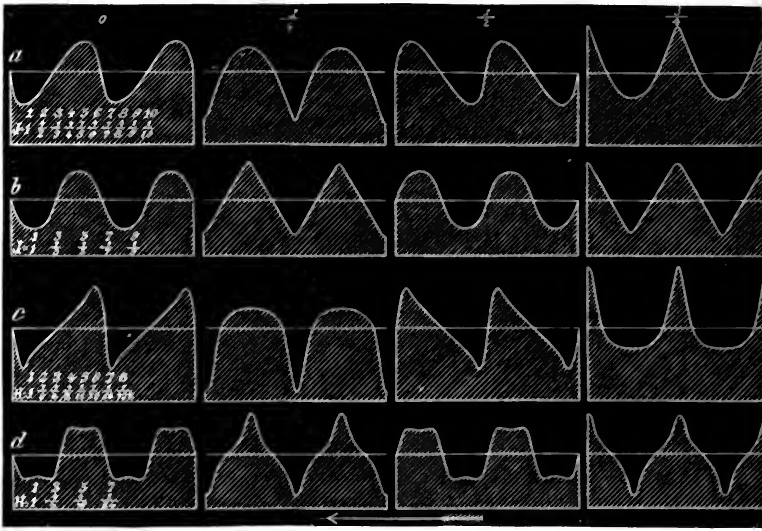


FIG. 81.

it gave the inferior beat-tone (256) plainly enough. Another bar giving 2048 and 3840 yielded 256 as the *superior* beat-tone. Hence the beat-tone is not due to any peculiarity resulting from the separate origins of the two wave-systems.

Any series of maxima and minima of sounds of any pitch, if isochronous and similar, produces a tone the pitch of which is due to the frequency of such maxima and minima. If, for example, the sound of a shrill whistle be interrupted 128 times in a second, a note of this frequency will be heard. Dr. Koenig used

a metal disc pierced with 16 holes, revolving at the rate of 8 times in a second: this cut off and admitted the sound of forks of all pitches from 256 to 4096, and the note C with 128 v.s. was always present. This also tends to confirm what has been said about the coalescence of beats, and we may remark in passing that it prevents the analysis of a compound tone being made by such means.

With regard to differences of phase affecting the ear, Koenig has attempted to show that they are not without effect, as follows. He draws the curve, compounded of a prime and its first ten harmonics, with continuously-diminishing amplitudes: in one case they all start in the same phase, in another all the partials are $\frac{1}{4}$ wave in advance of the prime, in another $\frac{1}{2}$ wave, and so on (Fig. 81). These curves are then cut out on a metal band, the ends of which are subsequently joined. Air is blown against the edge through a narrow slit, and it is found that the sounds produced differ according to the curve used. To look at the curves it would be strange if they did not; but whether the wave-systems emitted in air reproduce all the components as cut on the edge and no other is perhaps open to doubt. When the phase difference is $\frac{1}{4}$ the tone is loudest and most strident, with $\frac{3}{4}$ it is soft and gentle, and intermediate at 0 and $\frac{1}{2}$. The apparatus is called a Wave-Siren.

In another instrument (Fig. 82) the same principle is pursued, but in a different manner. A series of brass discs—16 in all—are mounted one behind the other on a common axle, the rims of these are cut out in simple harmonic curves of diminishing amplitude and period. Each can be blown by a jet of air coming from a narrow slit, and a keyboard enables the operator to set any or all of the jets in action at will. Differences of phase may be brought about by setting the discs at a small angle as compared with their original positions, and the slits themselves may be tilted, thus introducing a difference of the same kind. The results show that the sound does vary with the phase, being purer with a difference $= 0$, and more strident and nasal when it $= \frac{1}{2}$ (see Koenig's *Quelques Expériences d'Acoustique*, or *Nature*, vol. xxvi. pp. 204, 276).

Some of Helmholtz's views on upper partials also appear to

require modification. For example, in wires the periods of the smaller wave-motions superposed on the primary are not exact submultiples of its period, because the relative rigidity of

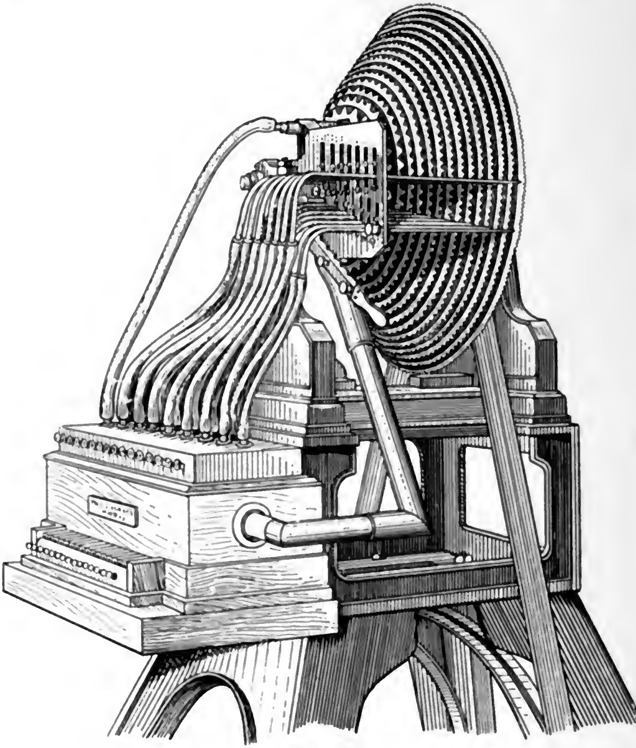


FIG. 82.

a wire for short lengths is greater than for long ones, and though Fourier's theorem is beyond dispute, it may not be strictly applicable as a matter of physics.

So, again, the use of resonators tuned to the theoretical upper

partials of organ pipes, vowel sounds, etc., tends to mislead, because the partials actually present are not of the theoretical pitch, and the synthesis of simple tones, *e.g.* those of tuning-forks, fails to give the compound tone of an instrument for a similar reason.

There can be no doubt of the value of Helmholtz's work, and of the enormous advance in the science of Acoustics which is due to him ; but here, as in other cases, it is not given to any one man to exhaust a scientific subject. (See also Koenig on the "Wave-Siren," *Wiedemann's Annalen*, lvii. p. 339, 1896.)

APPENDIX IV

The following is a list of some of the chief contributors to mathematical or experimental Acoustics, with dates. It is not exhaustive, and does not include the names of physicists who are happily still living. Most of them were Professors of Physics or Mathematics at British or foreign universities and colleges; perhaps the most singular exception was Scheibler, a silk merchant of Crefeld in Rhenish Prussia.

Arago, Dominique François, 1786-1853.
Bernouilli, Daniel,[1] 1700-1782.
Biot, Jean Baptiste, 1774-1862.
Boyle, Hon. Robert, 1627-1691.
Chladni, Ernst Florens Friedrich, 1756-1827.
Colladon, Jean Daniel, 1802-1893.
Corti, Bonaventura, 1729-1813.[2]
Despretz, César Mansuète, 1792-1863.
Donders, François Cornelius, 1818-1889.
Döppler, Chrétien, 1803-1854.
Dove, Heinrich Wilhelm, 1803-1879.
Duhamel, Jean Marc Constant, 1797-1872.
Dulong, Pierre Louis, 1785-1838.
Earnshaw, Rev. Samuel, 1805-1888.
Ellis, Alexander John, 1814-1890.
Euler, Leonard, 1707-1783.
Faraday, Michael, 1791-1867.
Foucault, Jean Bernard Léon, 1819-1868.
Fourier, Jean Baptiste Joseph, 1768-1830.
Galilei, Galileo, 1564-1642.
Gauss, Karl Friedrich, 1777-1855.
Green, George, 1793-1841.
Griesbach, John Henry, 1798-1875.

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[1] One member of a most distinguished family.

[2] According to both the *Century Dictionary* and the *New English Dictionary*, the fibres, arches, etc., of the cochlea take their name from Bonaventura Corti; but, on the other hand, Wernich and Hirsch (*Biographisches Lexikon*, 1884) speak of the Marquis Alfonso Corti, who "seinen Namen mit der Histologie der Gehörwerkzeuge durch das *nach ihm* benannte Corti'sche Organ für immer verknüpft." Probably the former is the more correct. See also p. 160.

Helmholtz, Hermann Ludwig Ferdinand von, 1821-1894.
Herschel, Sir John Frederick William, 1792-1871.
Hopkins, William, 1793-1866.
Kastner, Frederick, 1852-1882.
Kirchhoff, Gustav Robert, 1824-1887.
Kundt, August, 1838-1894.
Laplace, Pierre Simon, 1749-1827.
Latour, Baron Cagniard, 1777-1859.
Lissajous, Jules Antoine, 1822-1880.
Marloye, Albert, 1795-1874.
Mercadier, Paul Louis, 1812-1889.
Mersenne, Marin, 1588-1648.
Newton, Sir Isaac, 1642-1727.
Ohm, Georg Simon, 1787-1854.
Poisson, Simeon Denis, 1781-1840.
Regnault, Henri Victor, 1810-1878.
Reis, Philip, 1834-1874.
Saussure, Nicolas Theodore, 1767-1845.
Sauveur, Joseph, 1653-1716.
Savart, Felix, 1791-1841.
Scheibler, Johann Heinrich, 1777-1837.
Seebeck, Johann Thomas, 1770-1831.
Smith, Dr. Robert, 1689-1768.
Sondhauss, Karl Friedrich Julius, 1815-1886.
Sorge, Georg Andreas, 1703-1778.
Sturm, Jacques, 1803-1855.
Tartini, Giuseppe, 1692-1770.
Terquem, Olry, 1823-1888.
Weber, Ernst Heinrich, 1795-1878.
Weber, Guillaume Edouard, 1804-1891.
Wheatstone, Sir Charles, 1802-1875.
Willis, Rev. Robert, 1800-1875.
Wollaston, Dr. William Hyde, 1766-1828.
Young, Dr. Thomas, 1773-1829.

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THE END

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